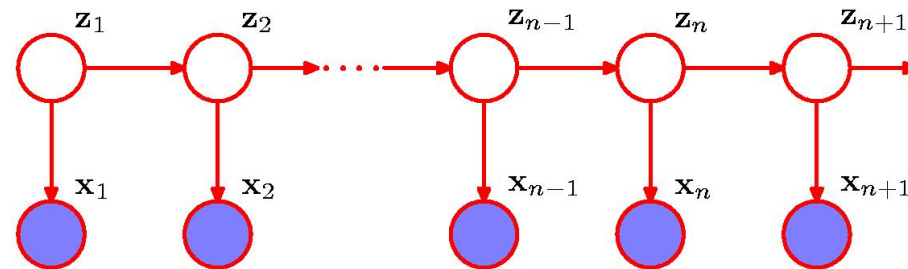


# Hidden Markov Models

Selecting the initial model parameters

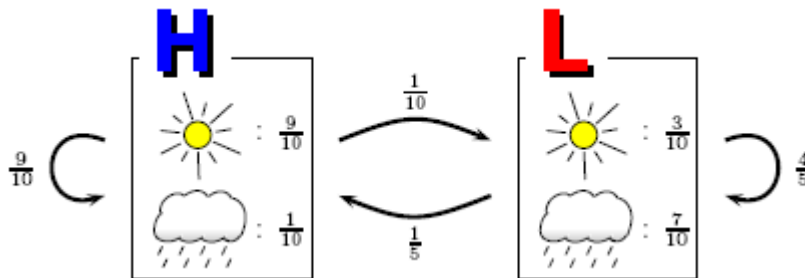
Using HMMs for (simple) gene finding



# HMMs as a generative model

A HMM *generates a sequence of observables* by moving from latent state to latent state according to the transition probabilities and *emitting an observable* (from a discrete set of observables, i.e. a finite alphabet) from each latent state visited *according to the emission probabilities* of the state ...

Model  $M$ :



A *run* follows a sequence of states:

H H L L H

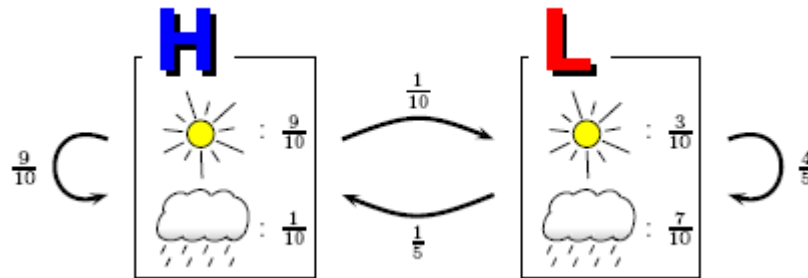
And *emits* a sequence of symbols:



For a HMM that generates finite strings (e.g. a HMM with an end-state), the language  $L = \{\mathbf{X} \mid p(\mathbf{X}) > 0\}$  is regular ...

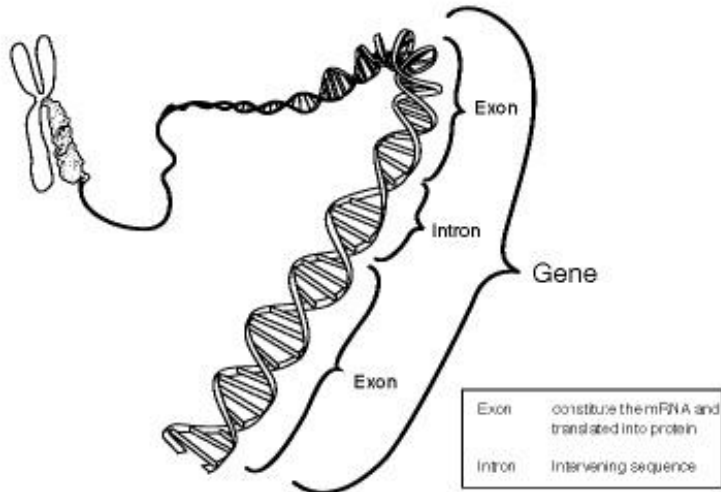
# Selecting initial model parameters

The initial selection of transition and emission probabilities, i.e.  $A$ ,  $\pi$ ,  $\Phi$ , should model (how we see) the underlying structure of the observations, i.e. the syntax of possible sequences of observations, recall that the language  $L = \{x \mid P(x \mid \theta) > 0\}$  is regular.



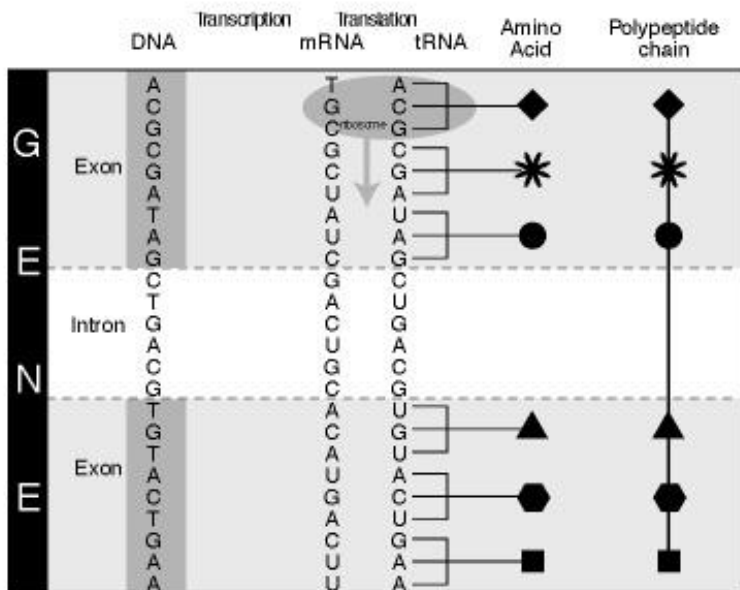
The initial selection of parameters is essential just to decide which parameters are 0 (or 1), i.e. to decide which transitions or emissions should never (or always) be possible ...

# Example – Gene finding



Each protein is encoded in a stretch of DNA. A **gene** ...

Which is **expressed** when the protein is needed ...



## Important problem

Locating genes on the genome and determining how they get expressed ...

Recognizing the patterns that indicates a gene ...

# GENETIC CODE CRACKED FULL STORY

| 1ST<br>↓ | 2ND<br>→ | U                           | C                        | A                            | G                         | 3RD<br>↓         |
|----------|----------|-----------------------------|--------------------------|------------------------------|---------------------------|------------------|
| U        |          | PHE<br>PHE<br>LEU<br>LEU    | SER<br>SER<br>SER<br>SER | TYR<br>TYR<br>Ochre<br>Amber | CYS<br>CYS<br>Opal<br>TRP | U<br>C<br>A<br>G |
| C        |          | LEU<br>LEU<br>LEU<br>LEU    | PRO<br>PRO<br>PRO<br>PRO | HIS<br>HIS<br>GLN<br>GLN     | ARG<br>ARG<br>ARG<br>ARG  | U<br>C<br>A<br>G |
| A        |          | ILEU<br>ILEU<br>ILEU<br>MET | THR<br>THR<br>THR<br>THR | ASPN<br>ASPN<br>LYS<br>LYS   | SER<br>SER<br>ARG<br>ARG  | U<br>C<br>A<br>G |
| G        |          | VAL<br>VAL<br>VAL<br>VAL    | ALA<br>ALA<br>ALA<br>ALA | ASP<br>ASP<br>GLU<br>GLU     | GLY<br>GLY<br>GLY<br>GLY  | U<br>C<br>A<br>G |

PHE - PHENYLALANINE  
 GLU - GLUTAMIC ACID  
 ASP - ASPARTIC ACID  
 ASPN - ASPARAGINE  
 ILEU - ISOLEUCINE  
 MET - METHIONINE  
 THR - THREONINE  
 ARG - ARGinine  
 GLN - GLUTAMINE  
 HIS - HISTIDINE  
 TRP - TRYPTOPHAN  
 TYR - TYROSINE  
 CYS - CYSTEINE  
 LEU - LEUCINE  
 PRO - PROLINE  
 ALA - ALANINE  
 VAL - VALINE  
 GLY - GLYCINE  
 LYS - LYSINE  
 SER - SERINE

**KEY**

Here it is. The code for each of the twenty amino acids. So simple isn't it? Read the table and you can't be wrong.

>NC\_002737.1 Streptococcus pyogenes M1 GAS

TTGTTGATATTCTGTTTTTCTTTTTTAGTTTTCCACATGAAAAATAGTTGAAAACAATA  
GCGGTGTCCCCTTAAAATGGCTTTTCCACAGGTTGTGGAGAACCCAAATTAACAGTGTTA  
ATTTATTTTCCACAGGTTGTGGAAAACTAACTATTATCCATCGTTCTGTGGAAAACTAG  
AATAGTTTATGGTAGAATAGTTCTAGAATTATCCACAAGAAGGAACCTAGTATGACTGAA  
AATGAACAAATTTTTTGGAACAGGGTCTTGGAATTAGCTCAGAGTCAATTAACAGGCA  
ACTTATGAATTTTTTGTTCATGATGCCCCTCTATTAAGGTCGATAAGCATATTGCAACT  
ATTTACTTAGATCAAATGAAAGAGCTCTTTTGGGAAAAAATCTTAAAGATGTTATTCTT  
ACTGCTGGTTTTGAAGTTTATAACGCTCAAATTTCTGTTGACTATGTTTTCGAAGAAGAC  
CTAATGATTGAGCAAAATCAGACCAAAATCAACCAAAAACCTAAGCAGCAAGCCTTAAAT  
TCTTTGCCTACTGTTACTTCAGATTTAACTCGAAATATAGTTTTGAAAACCTTTATTCAA  
GGAGATGAAAATCGTTGGGCTGTTGCTGCTTCAATAGCAGTAGCTAATACTCCTGGAAC  
ACCTATAATCCTTTGTTTATTTGGGGTGGCCCTGGGCTTGGAACCCATTTATTAAT  
GCTATTGGTAATTCTGTACTATTAGAAAATCCAAATGCTCGAATTAAATATATCACAGCT  
GAAAACCTTTATTAATGAGTTTGTTATCCATATTCGCCTTGATACCATGGATGAATTGAAA  
GAAAAATTTTCGAATTTAGATTTACTCCTTATTGATGATATCCAATCTTTAGCTAAAAAA  
ACGCTCTCTGGAACACAAGAAGAGTTCTTTAATACTTTTAAATGCACTTCATAATAAAC  
AAACAAATTGTCCTAACAAGCGACCGTACACCAGATCATCTCAATGATTTAGAAGATCGA  
TTAGTTACTCGTTTTAAATGGGGATTAAACAGTCAATATCACACCTCCTGATTTTGAAACA  
CGAGTGGCTATTTTGACAAATAAAATTCAAGAATATAACTTTATTTTTCCTCAAGATACC  
ATTGAGTATTTGGCTGGTCAATTTGATTTCTAATGTCAGAGATTTAGAAGGTGCCTTAAAA  
GATATTAGTCTGGTTGCTAATTTCAAACAAATTGACACGATTACTGTTGACATTGCTGCC  
GAAGCTATTCGCGCCAGAAAGCAAGATGGACCTAAAATGACAGTTATTCCCATCGAAGAA  
ATTCAAGCGCAAGTTGGAAAATTTACGGTGTTACCGTCAAAGAAATTAAGCTACTAAA  
CGAACACAAAATATTGTTTTAGCAAGACAAGTAGCTATGTTTTTAGCACGTGAAATGACA  
GATAACAGTCTTCCTAAAATTGGAAAAGAATTTGGTGGCAGAGACCATTCAACAGTACTC  
CATGCCTATAATAAAATCAAAAACATGATCAGCCAGGACGAAAGCCTTAGGATCGAAATT  
GAAACCATAAAAAACAAAATTAATAACATGTGGAAGAATATCTTTTATGAAATAGTT  
ATCCACAAGTTGTGAACATCCATTTAGTCTTGGATTCTCTCGTTTATTTAGAGTTATCCA  
CTATATACACAAGACCTACTACTACTATTATTATACTTATTAAATAAAGGAGTTCT

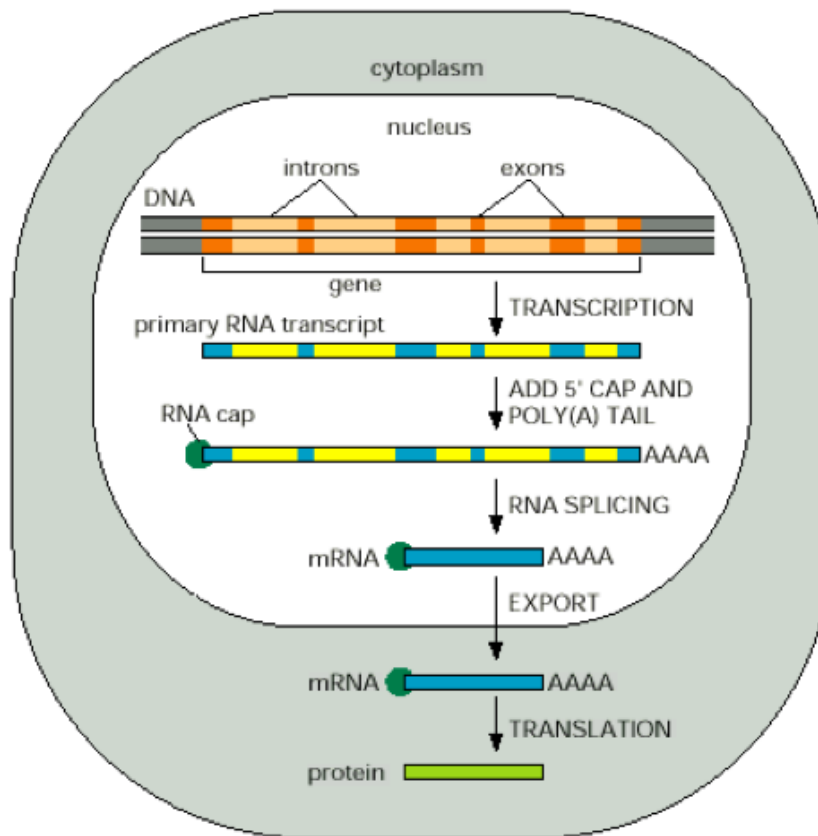
[illegible]

# Design a HMM that models the syntax of genes

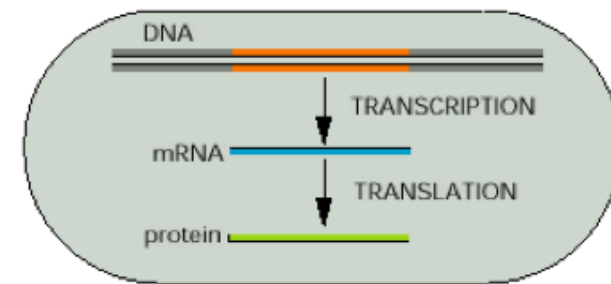
# Gene structure

Depends on the organism (eucaryote or procaryote)

(A) EUCARYOTES



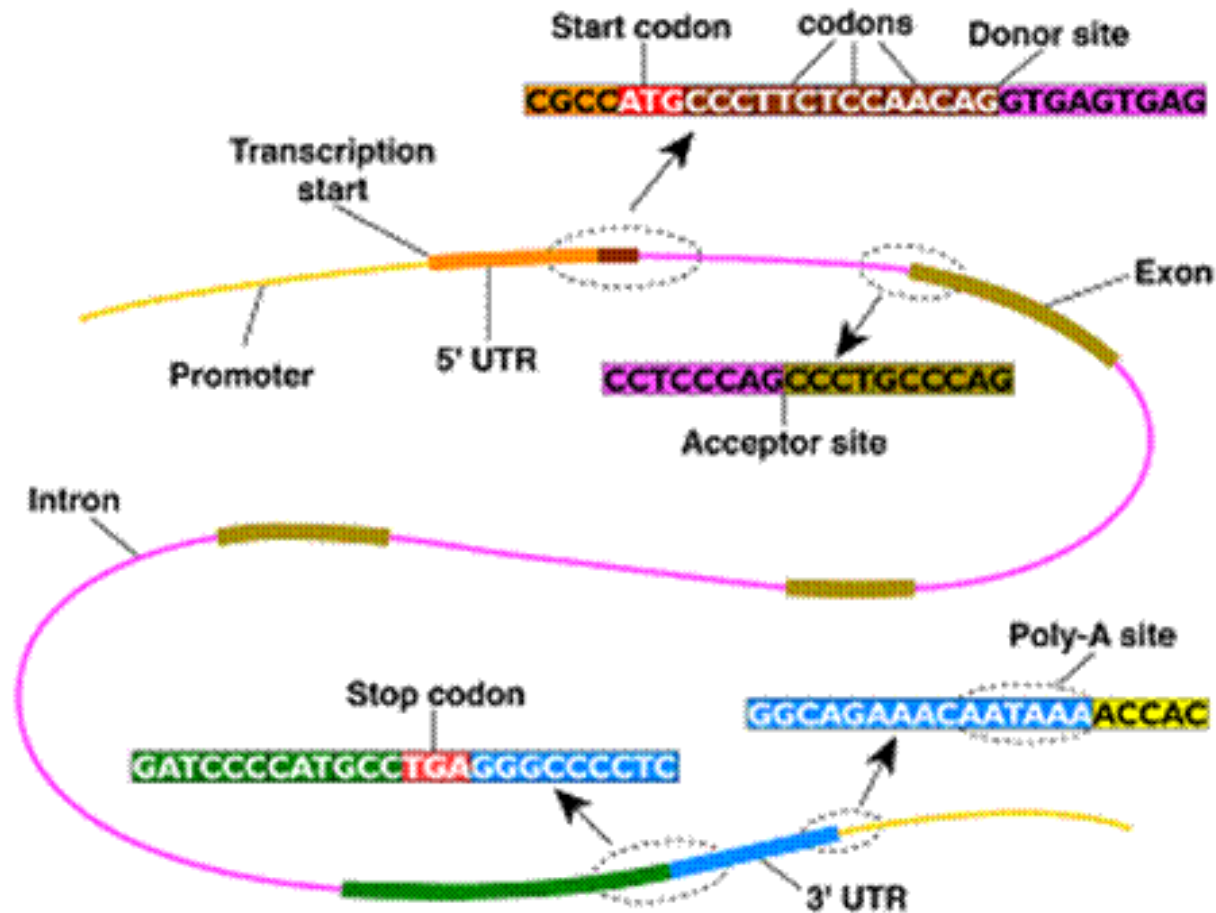
(B) PROCARYOTES



Smaller genomes and high coding density.

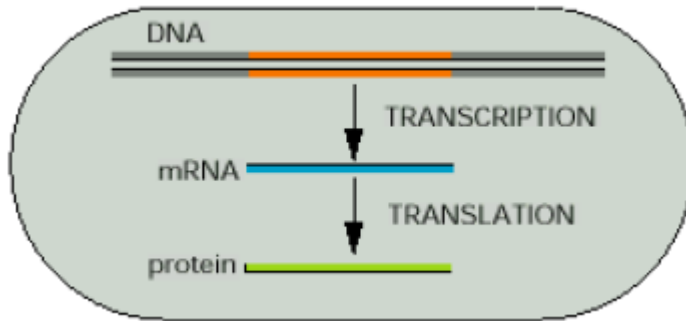
Large genomes. Intron/exon structure and low coding density

# Gene structure in eukaryotes



Eukaryotic gene structure in more details

# Gene structure in procaryotes

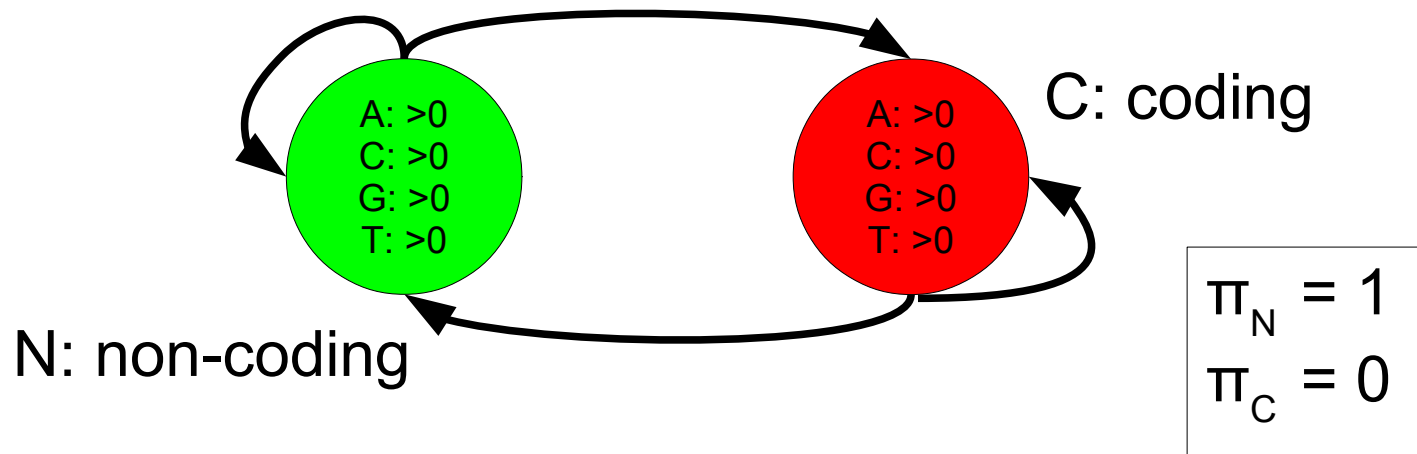


## Biological facts

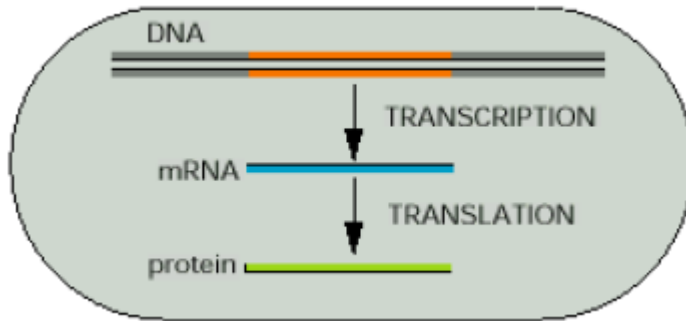
- The gene is a substring of the DNA sequence of A,C,G,T's

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

**X:** acgatgcgctaatatgtccgatgacgtgagcataagcgacatgcag



# Gene structure in procaryotes

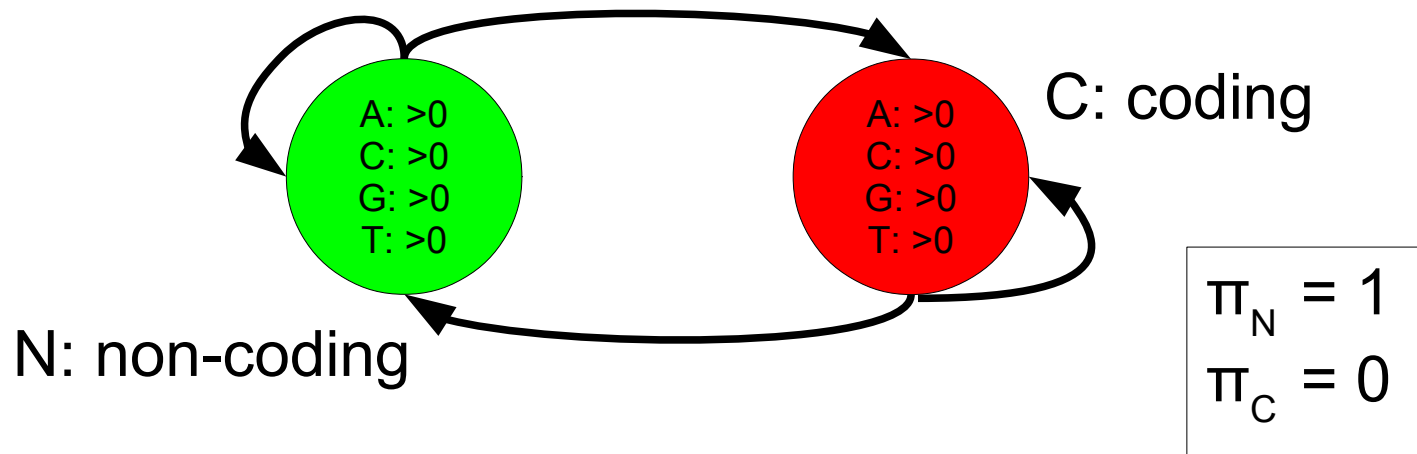


## Biological facts

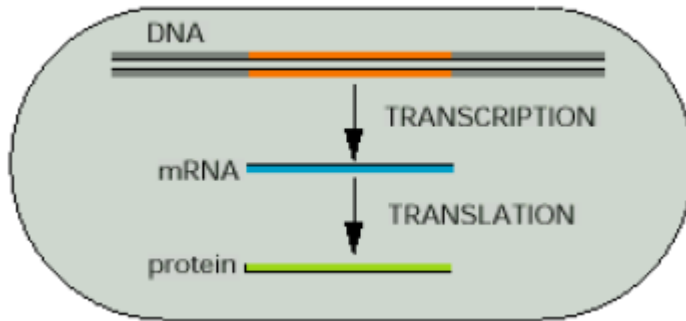
- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

**X:** acgatgcgctaatatgtccgatgacgtgagcataagcgacatgcag



# Gene structure in procaryotes



## Biological facts

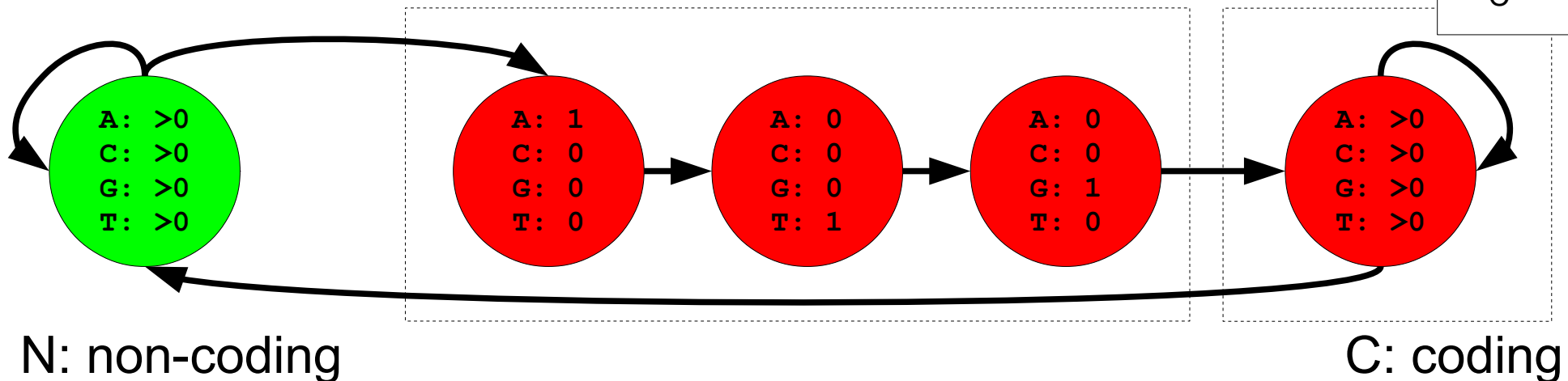
- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

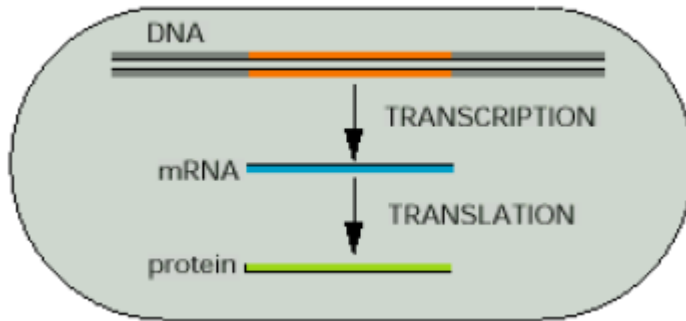
**X:** acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$

$$\pi_C = 0$$



# Gene structure in procaryotes



## Biological facts

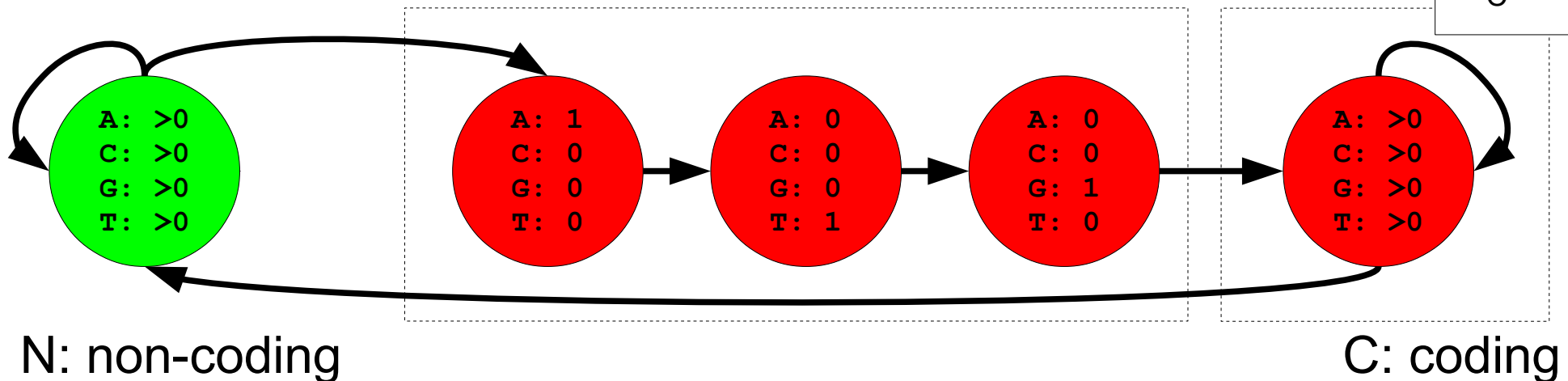
- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

**X:** acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$

$$\pi_C = 0$$

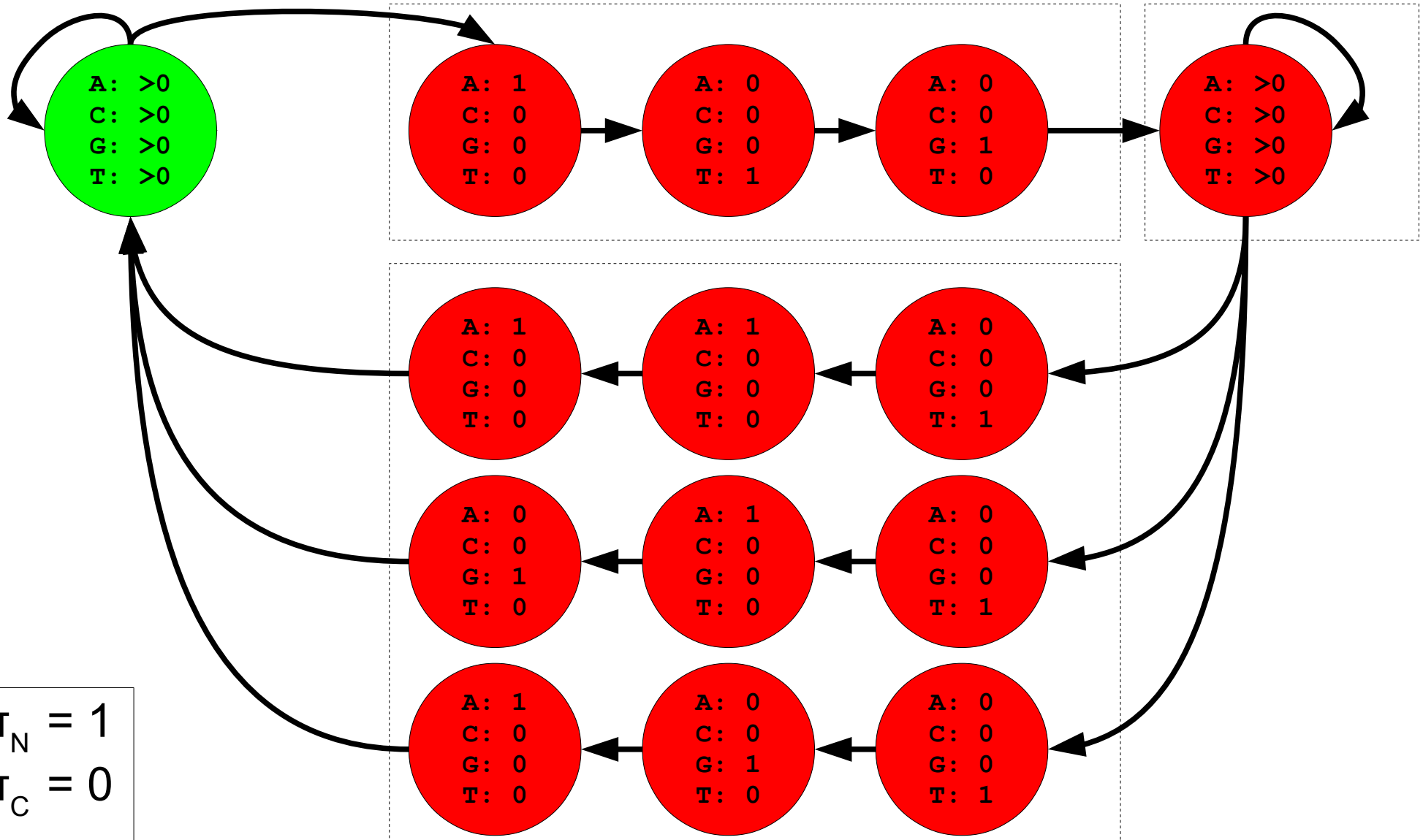


# Gene structure

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**

N: non-coding

C: coding

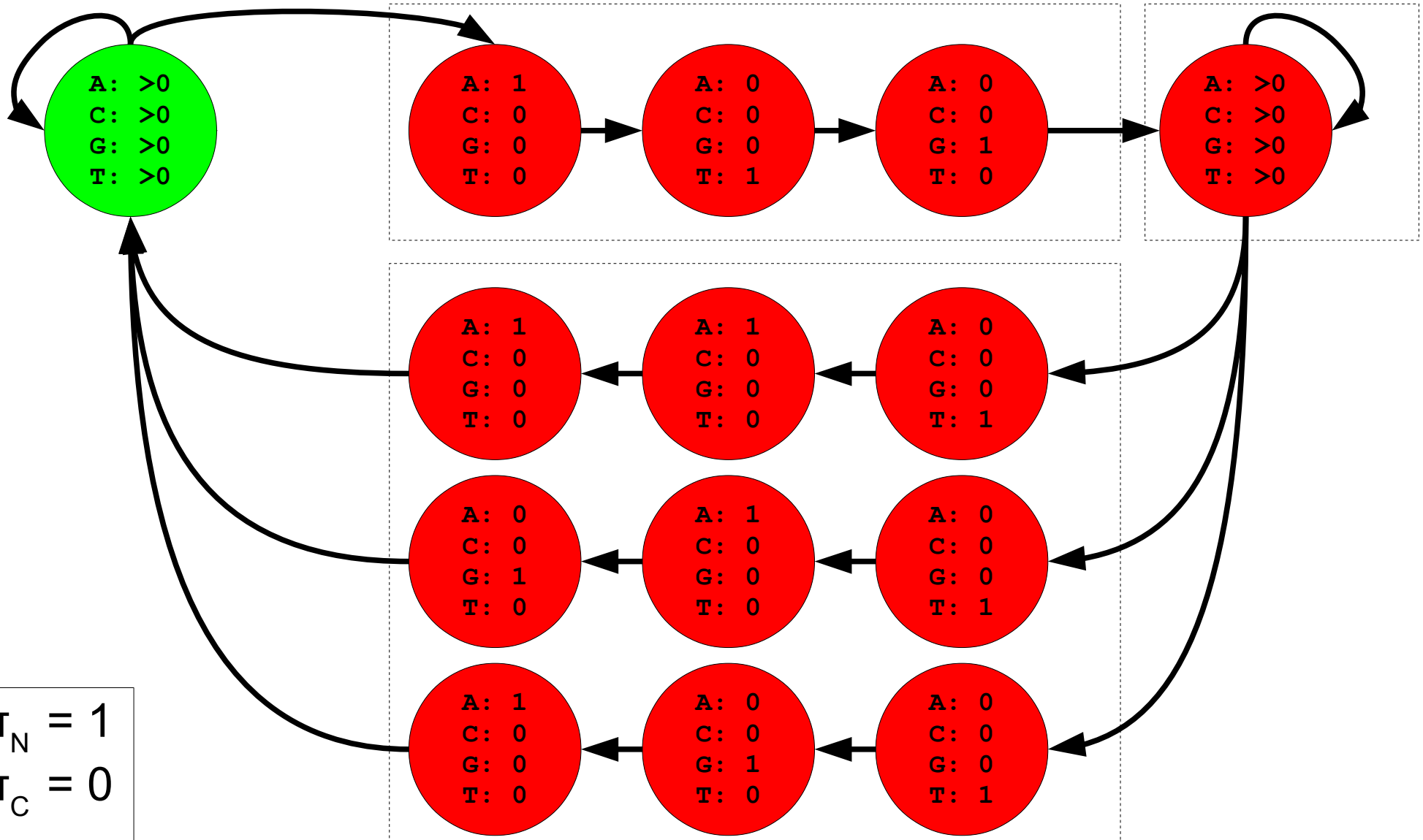


# Gene structure

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**
- The number of nucleotides in a gene is a multiplum of 3

N: non-coding

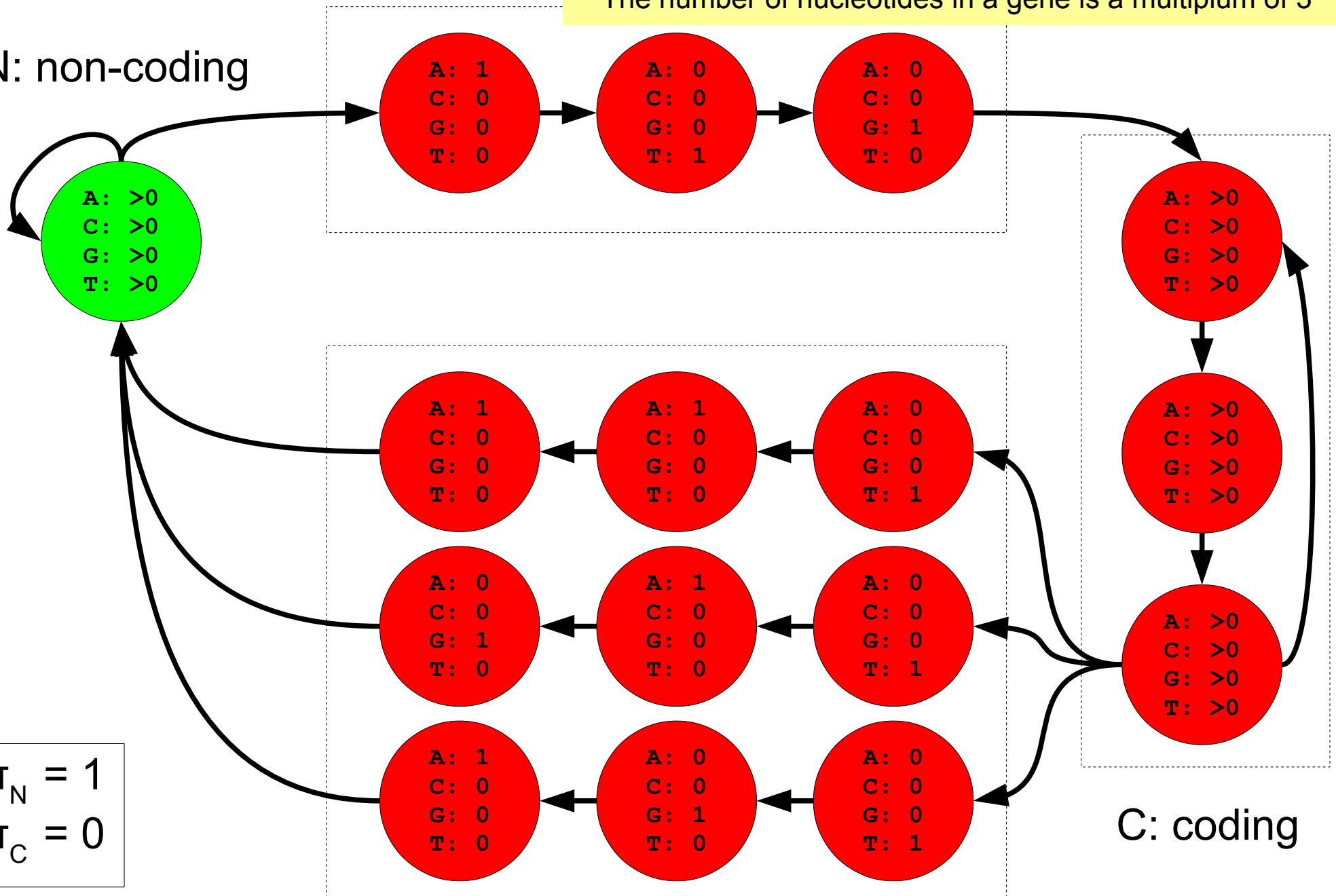
C: coding



# Gene structure

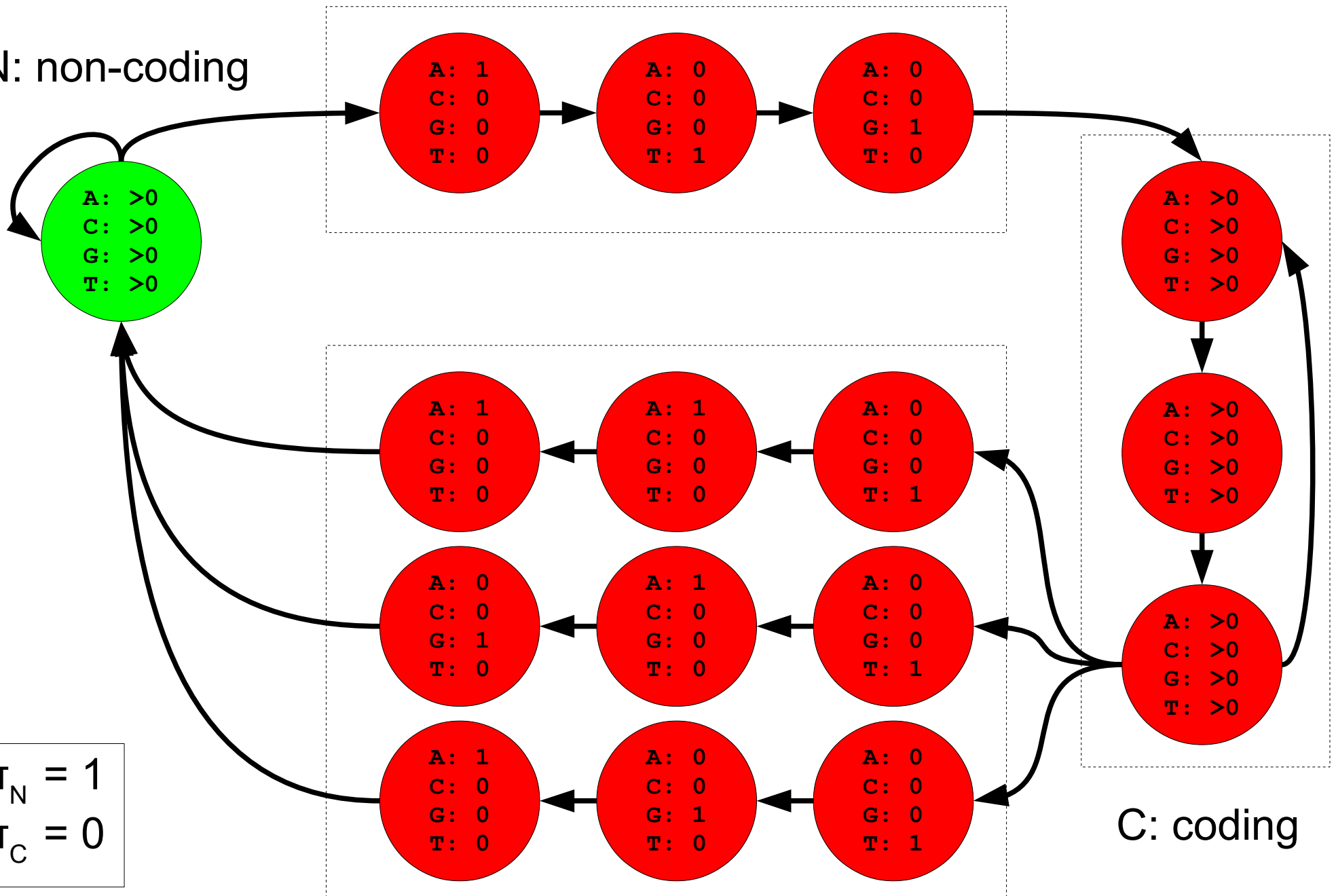
- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**
- The number of nucleotides in a gene is a multiplum of 3

N: non-coding

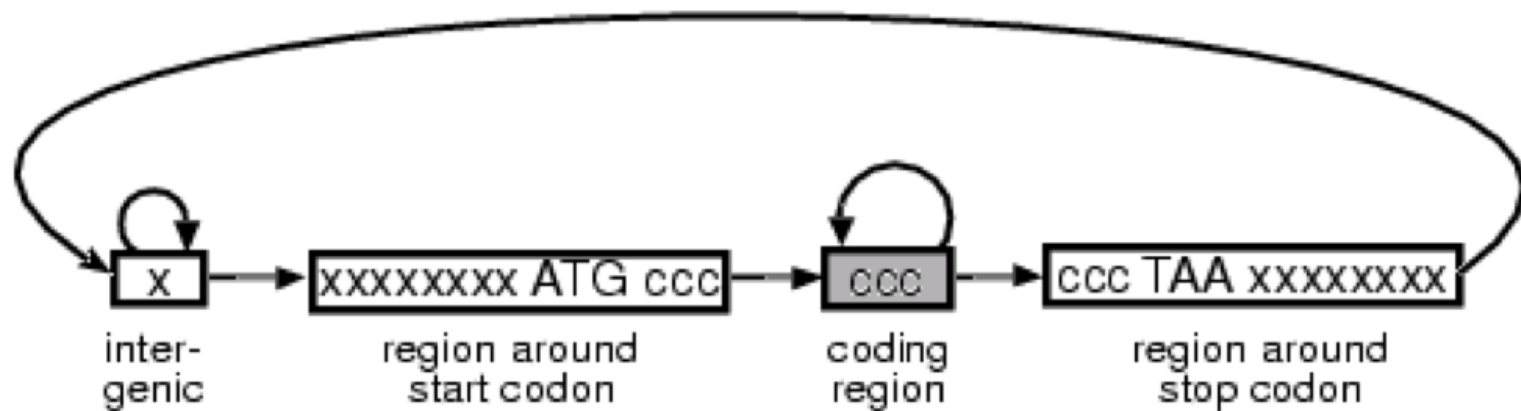


# Gene structure in procaryotes

N: non-coding



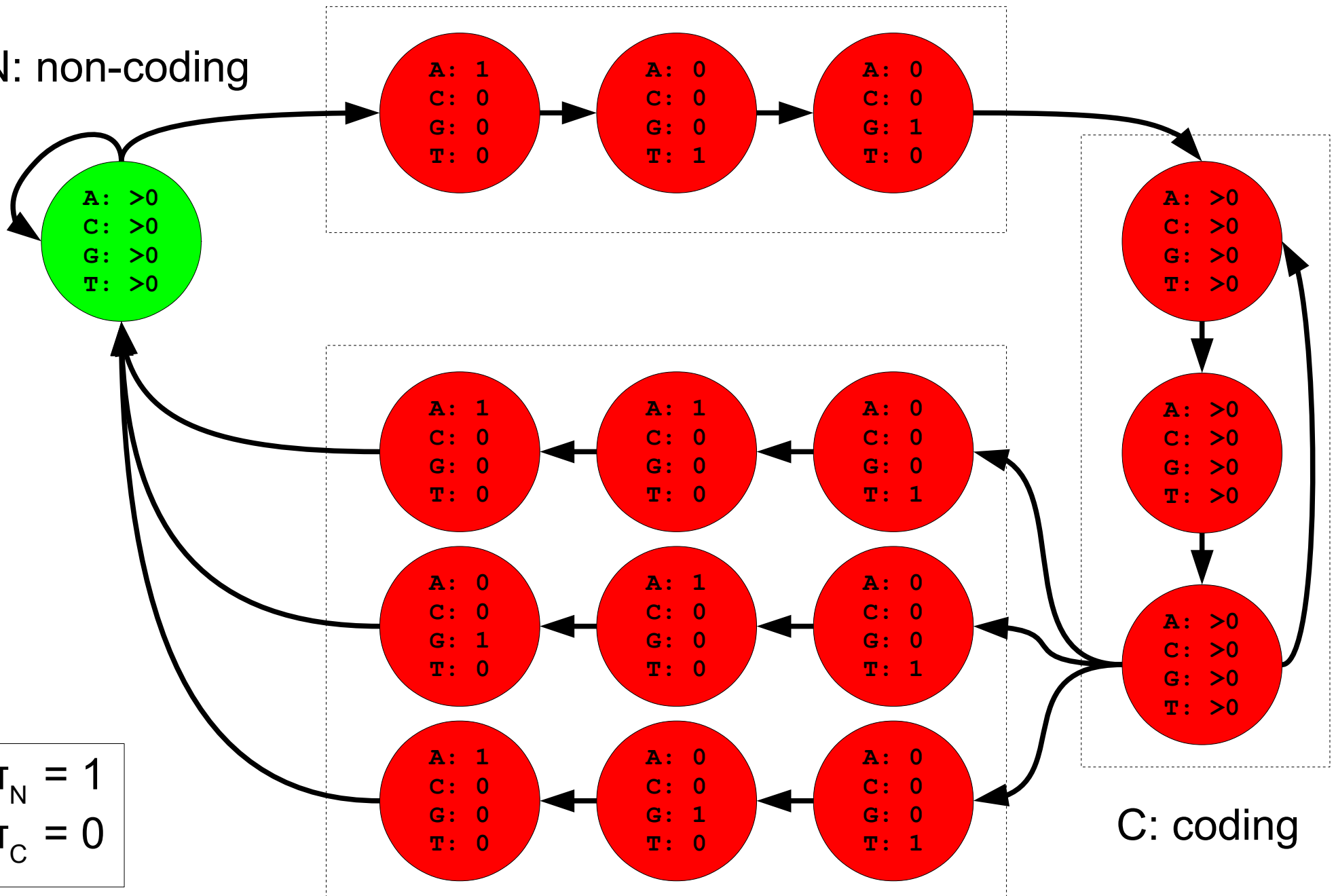
# Gene structure in procaryotes



From "An Introduction to HMMs for Biological Sequences", A. Krogh, 1998

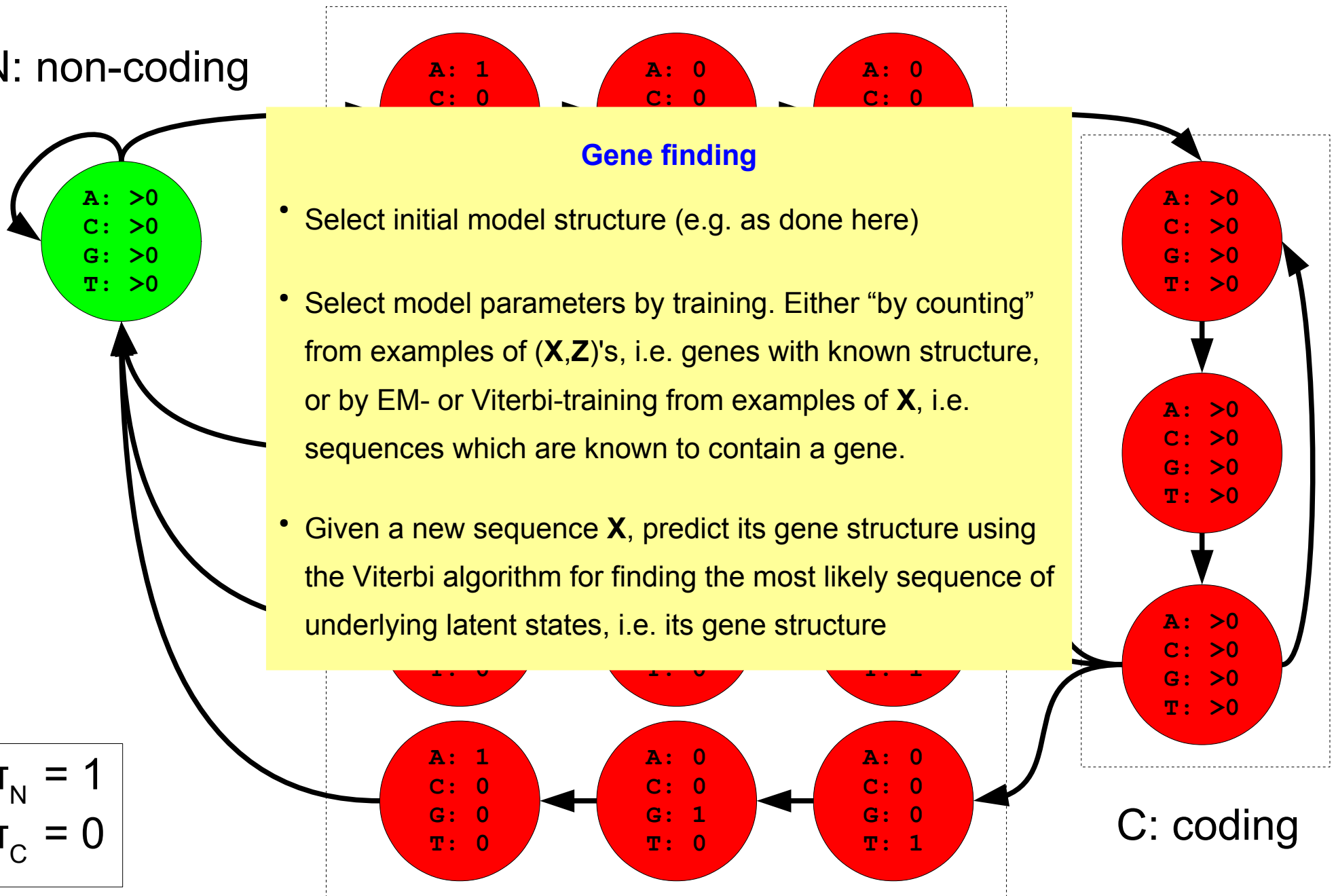
# Gene structure in procaryotes

N: non-coding



# Gene structure in procaryotes

N: non-coding



# Example – Gene finding

N: non-coding

A: 1  
C: 0

A: 0  
C: 0

A: 0  
C: 0

## Gene finding

- Select initial model structure (e.g. as done here)
- Select model parameters by training. Either “by counting” from examples of  $(\mathbf{X}, \mathbf{Z})$ 's, i.e. genes with known structure, or by EM- or Viterbi-training from examples of  $\mathbf{X}$ , i.e. sequences which are known to contain a gene.

## Even more biology

- There can be genes in both directions (and over lapping)



- There are more possible start-codons **atg**, **gtg**, and **tgg**
- Internal codons cannot be start- or stop-codons
- And a lot more ...

A: >0  
C: >0  
G: >0  
T: >0

A: >0  
C: >0  
G: >0  
T: >0

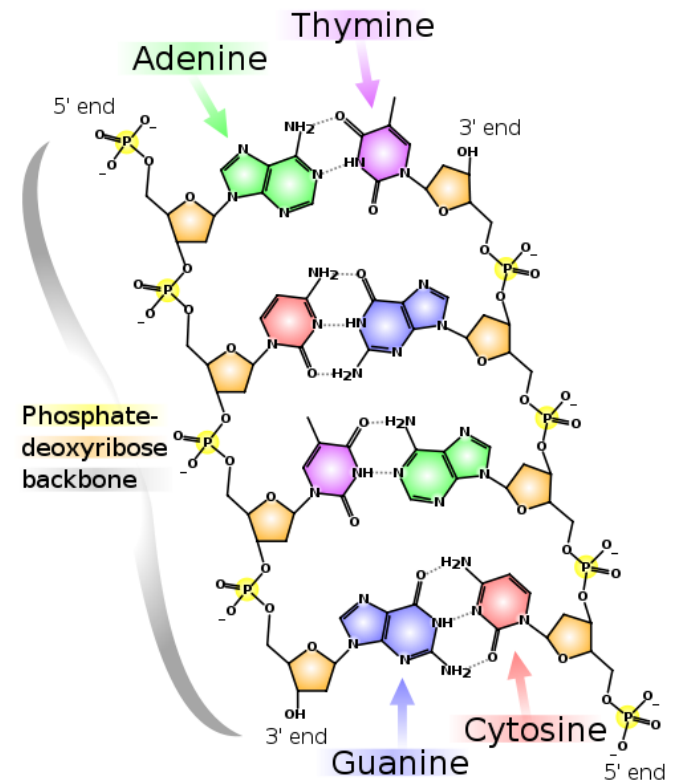
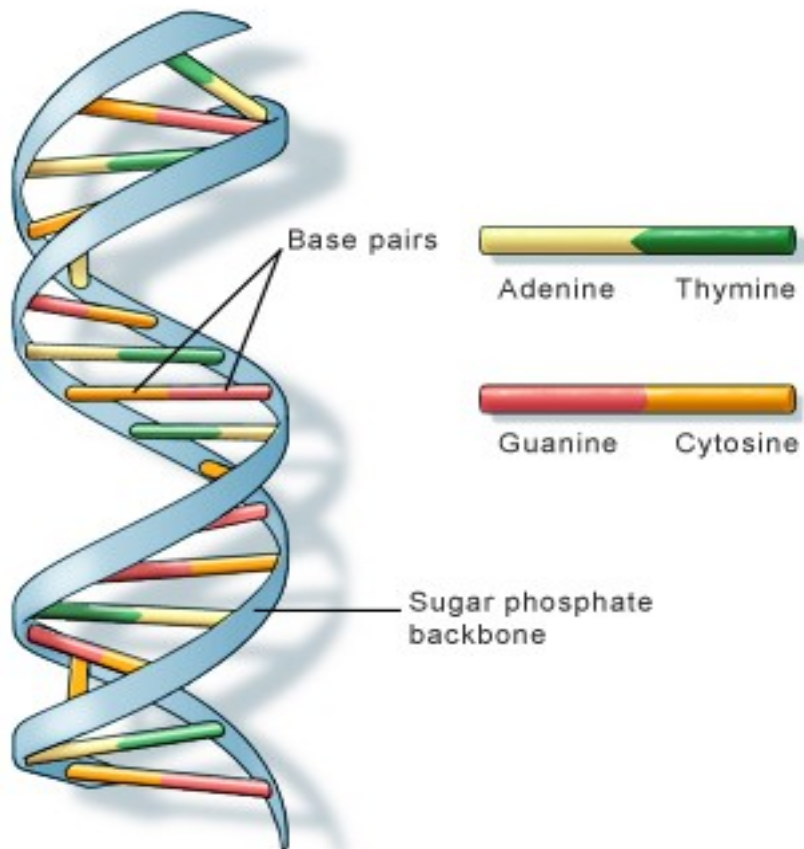
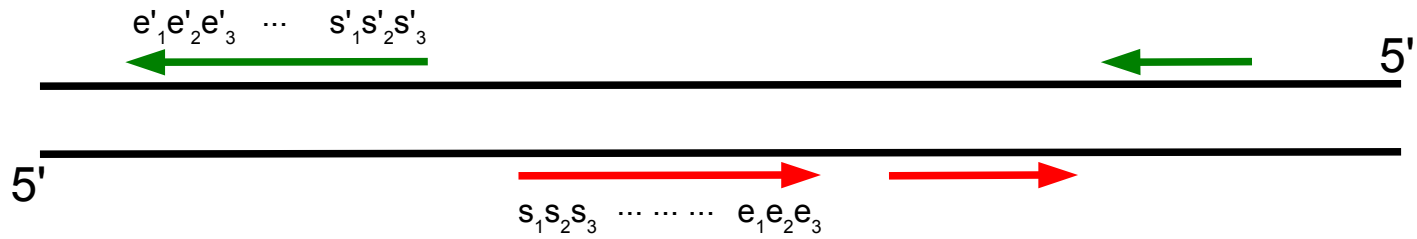
A: >0  
C: >0  
G: >0  
T: >0

C: coding

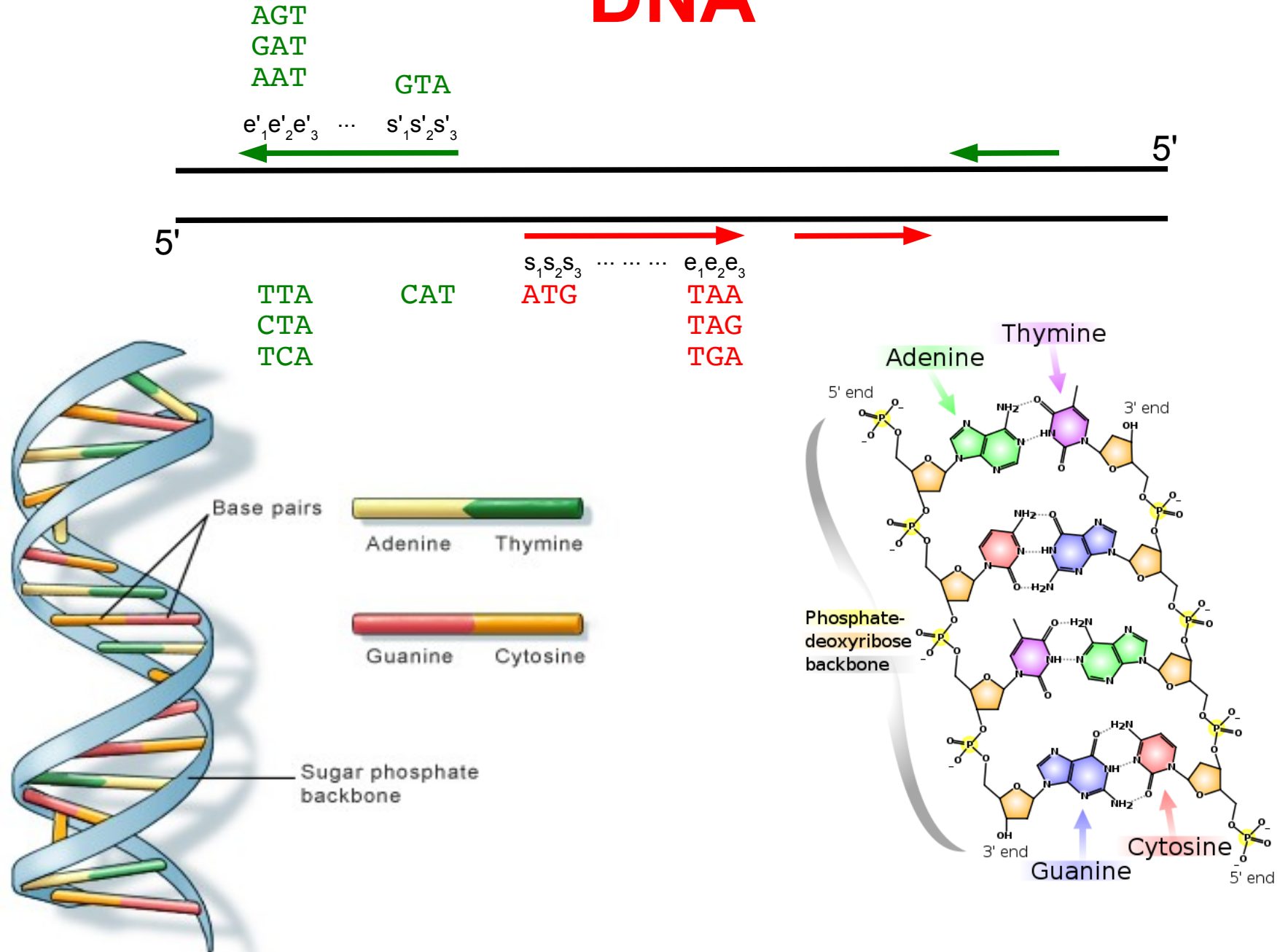
$$\pi_N = 1$$

$$\pi_C = 0$$

# DNA

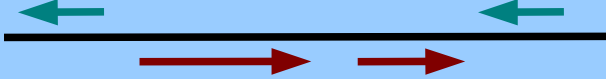


# DNA



## Even more biology

There can be genes in both directions



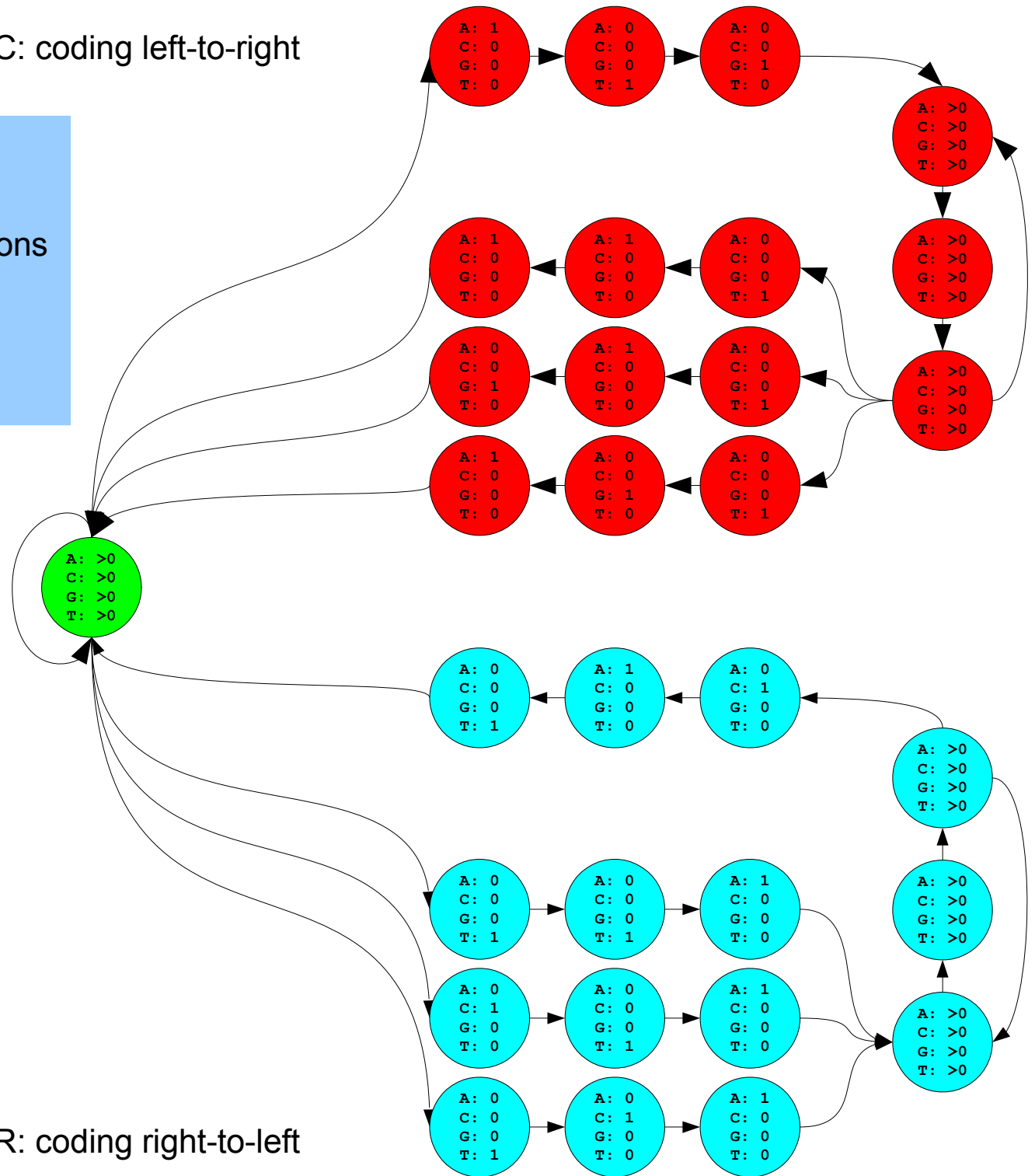
N: Non-coding

C: coding left-to-right

R: coding right-to-left

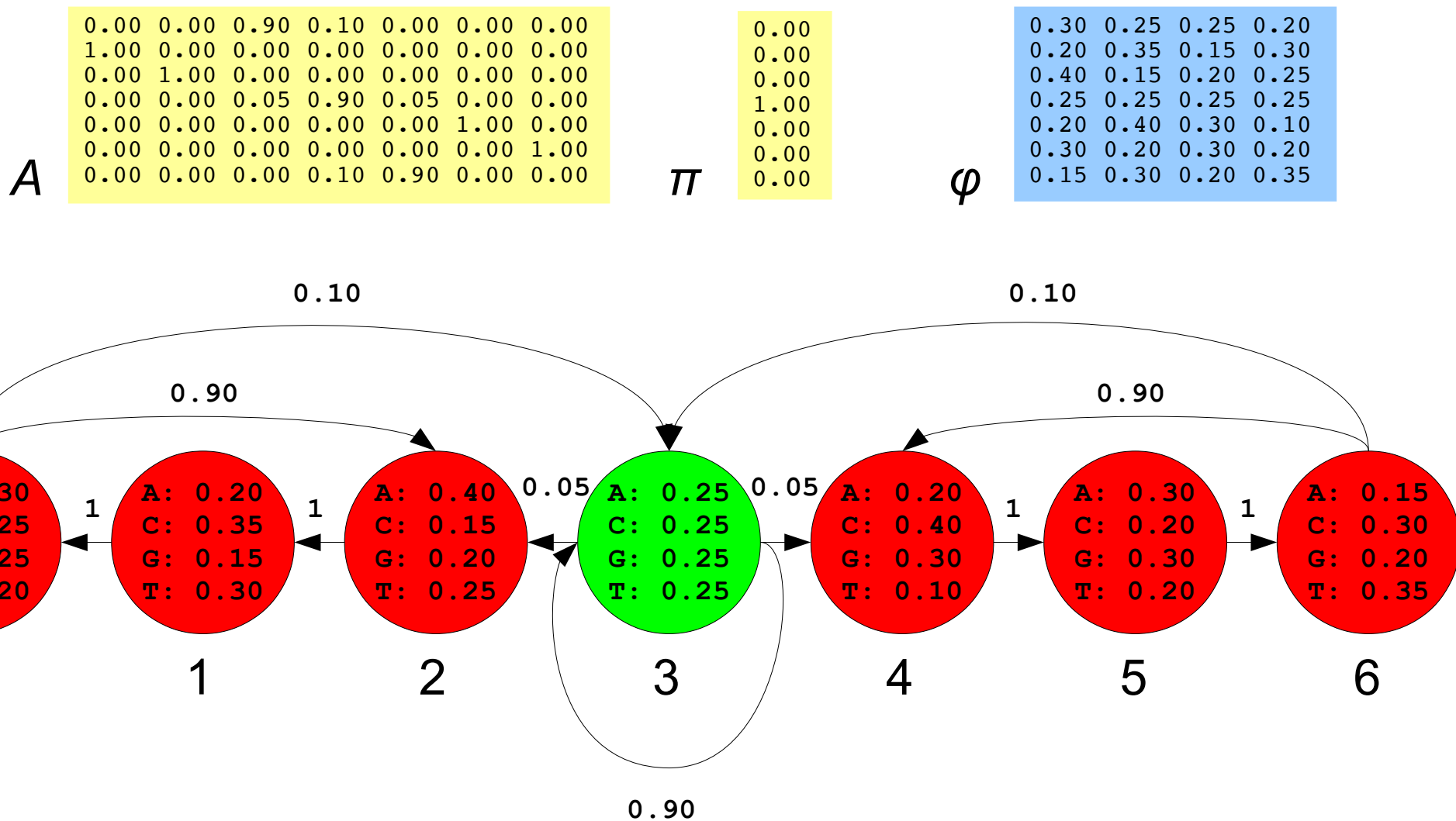
$$\pi_N = 1$$

$$\pi_C = 0$$



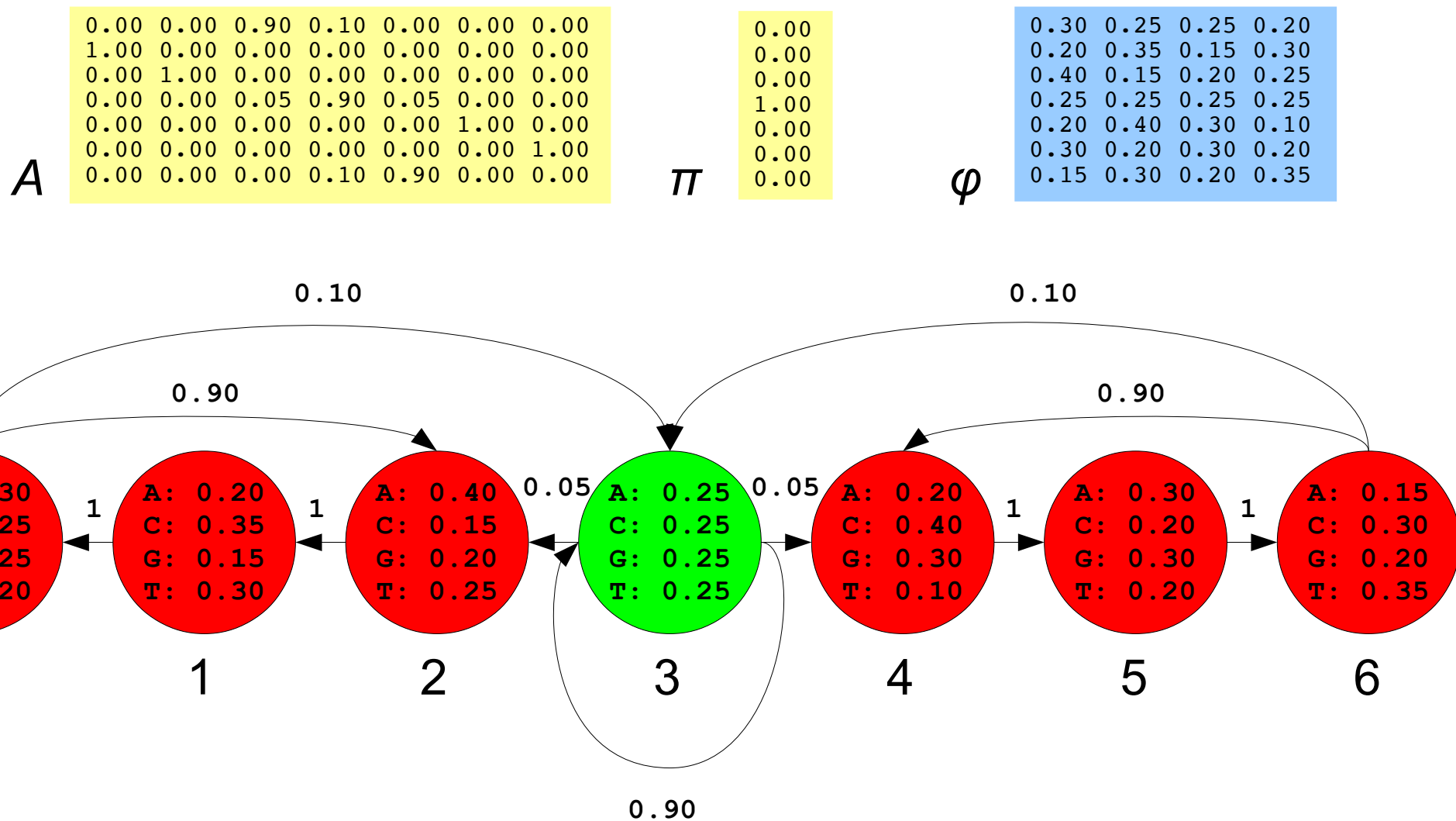
# Example – 7-state HMM

Observable: {A, C, G, T}, States: {0, 1, 2, 3, 4, 5, 6}

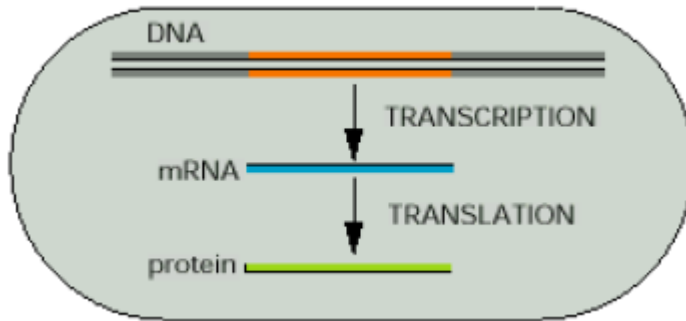


This model is also applicable for gene finding.

It does not model start- and stop-codons explicitly, but models that genes in both directions are a sequence of triplets.



# Problem: From annotation to Z



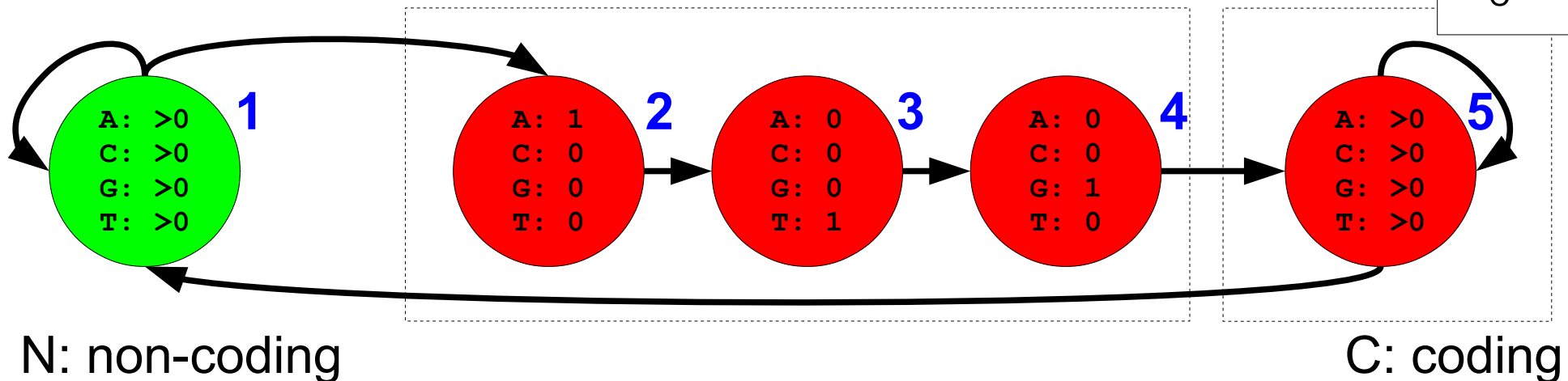
## Biological facts

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

**X:** acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$
$$\pi_C = 0$$



# Problem: From annotation to Z

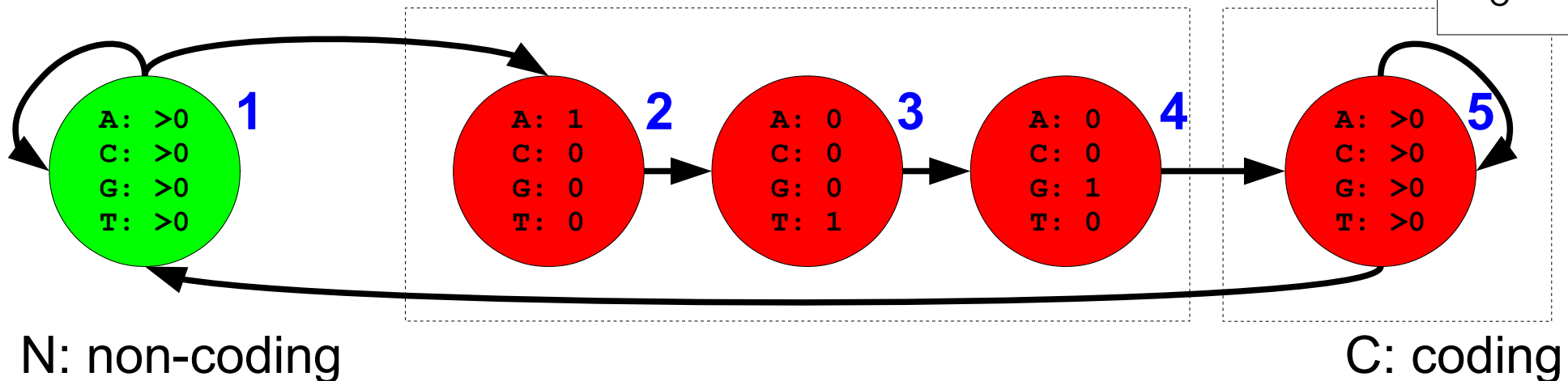
**Problem:** The string **Z**=NNNCCC.... is not a proper sequence of states in the illustrated HMM, but it can easily be converted into one (because there is a 1-1 matching between a sequence of Ns and Cs and a sequence of states).

ence of A,C,G,T's

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

**X:** acgatgcgctaataatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$
$$\pi_C = 0$$



# Problem: From annotation to Z

**Problem:** The string **Z**=NNNCCC.... is not a proper sequence of states in the illustrated HMM, but it can easily be converted into one (because there is a 1-1 matching between a sequence of Ns and Cs and a sequence of states).

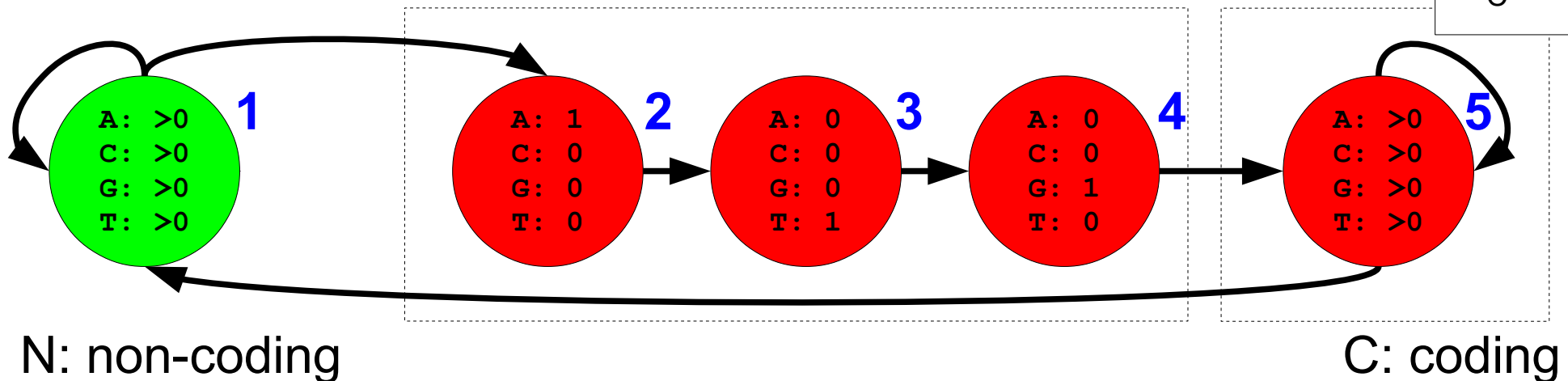
ence of A,C,G,T's

1112345555511111111234555555555555511111111111

**Z:** NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

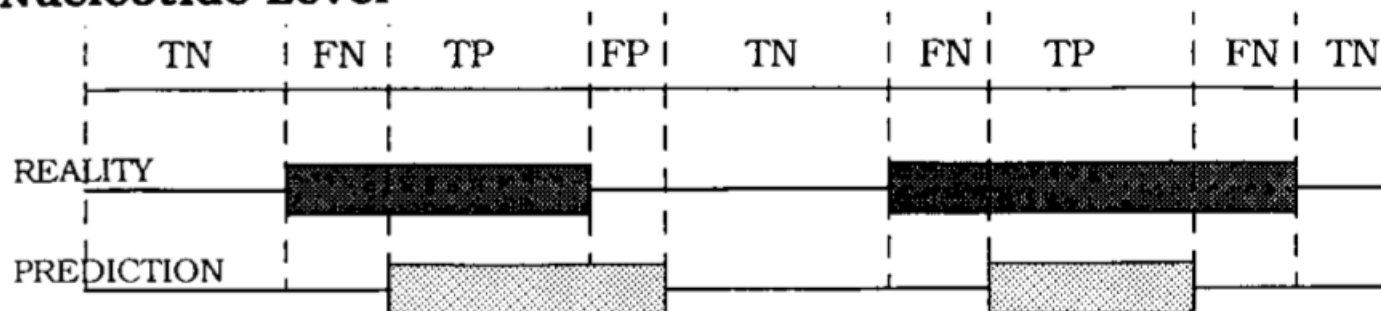
**X:** acgatgcgctaataatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$
$$\pi_C = 0$$



# Evaluating performance

## Nucleotide Level



|            |           | REALITY |           |       |
|------------|-----------|---------|-----------|-------|
|            |           | coding  | no coding |       |
| PREDICTION | coding    | TP      | FP        | TP+FP |
|            | no coding | FN      | TN        | FN+TN |
|            |           | TP+FN   | TN+FP     |       |

$$S_n = \frac{TP}{TP + FN}$$

**Sensitivity**

$$S_p = \frac{TN}{TN + FP}$$

**Specificity**

$$CC = \frac{(TP \times TN) - (FN \times FP)}{\sqrt{(TP + FN) \times (TN + FP) \times (TP + FP) \times (TN + FN)}}$$

**Correlation Coefficient**

$$ACP = \frac{1}{4} \left[ \frac{TP}{TP + FN} + \frac{TP}{TP + FP} + \frac{TN}{TN + FP} + \frac{TN}{TN + FN} \right]$$

$$AC = (ACP - 0.5) \times 2$$

**Approximate Correlation**

# compare\_anns.py

Genome 6

Cs (tp=757332, fp=164766, tn=305197, fn=57217): Sn = 0.9298, Sp = 0.8213, AC = 0.6213

Rs (tp=715865, fp=127462, tn=304830, fn=57584): Sn = 0.9255, Sp = 0.8489, AC = 0.6603

Both (tp=1473197, fp=292228, tn=247613, fn=114801): Sn = 0.9277, Sp = 0.8345, AC = 0.4520

Genome 7

Cs (tp=868820, fp=236008, tn=517048, fn=79049): Sn = 0.9166, Sp = 0.7864, AC = 0.6285

Rs (tp=815026, fp=226580, tn=511963, fn=84134): Sn = 0.9064, Sp = 0.7825, AC = 0.6205

Both (tp=1683846, fp=462588, tn=432914, fn=163183): Sn = 0.9117, Sp = 0.7845, AC = 0.4529

Genome 8

Cs (tp=705403, fp=137180, tn=351159, fn=74782): Sn = 0.9041, Sp = 0.8372, AC = 0.6424

Rs (tp=607762, fp=169829, tn=351738, fn=74203): Sn = 0.8912, Sp = 0.7816, AC = 0.5865

Both (tp=1313165, fp=307009, tn=276956, fn=148985): Sn = 0.8981, Sp = 0.8105, AC = 0.4166

Genome 9

Cs (tp=776640, fp=203664, tn=340882, fn=88415): Sn = 0.8978, Sp = 0.7922, AC = 0.5550

Rs (tp=759048, fp=219786, tn=336181, fn=93116): Sn = 0.8907, Sp = 0.7755, AC = 0.5270

Both (tp=1535688, fp=423450, tn=247766, fn=181531): Sn = 0.8943, Sp = 0.7839, AC = 0.3122

Genome 10

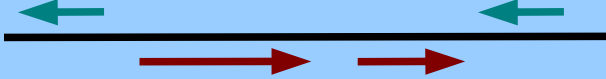
Cs (tp=612457, fp=106124, tn=253878, fn=88014): Sn = 0.8744, Sp = 0.8523, AC = 0.5872

Rs (tp=371869, fp=138143, tn=291605, fn=50287): Sn = 0.8809, Sp = 0.7291, AC = 0.5707

Both (tp=984326, fp=244267, tn=203591, fn=138301): Sn = 0.8768, Sp = 0.8012, AC = 0.3640

## Even more biology

There can be genes in both directions



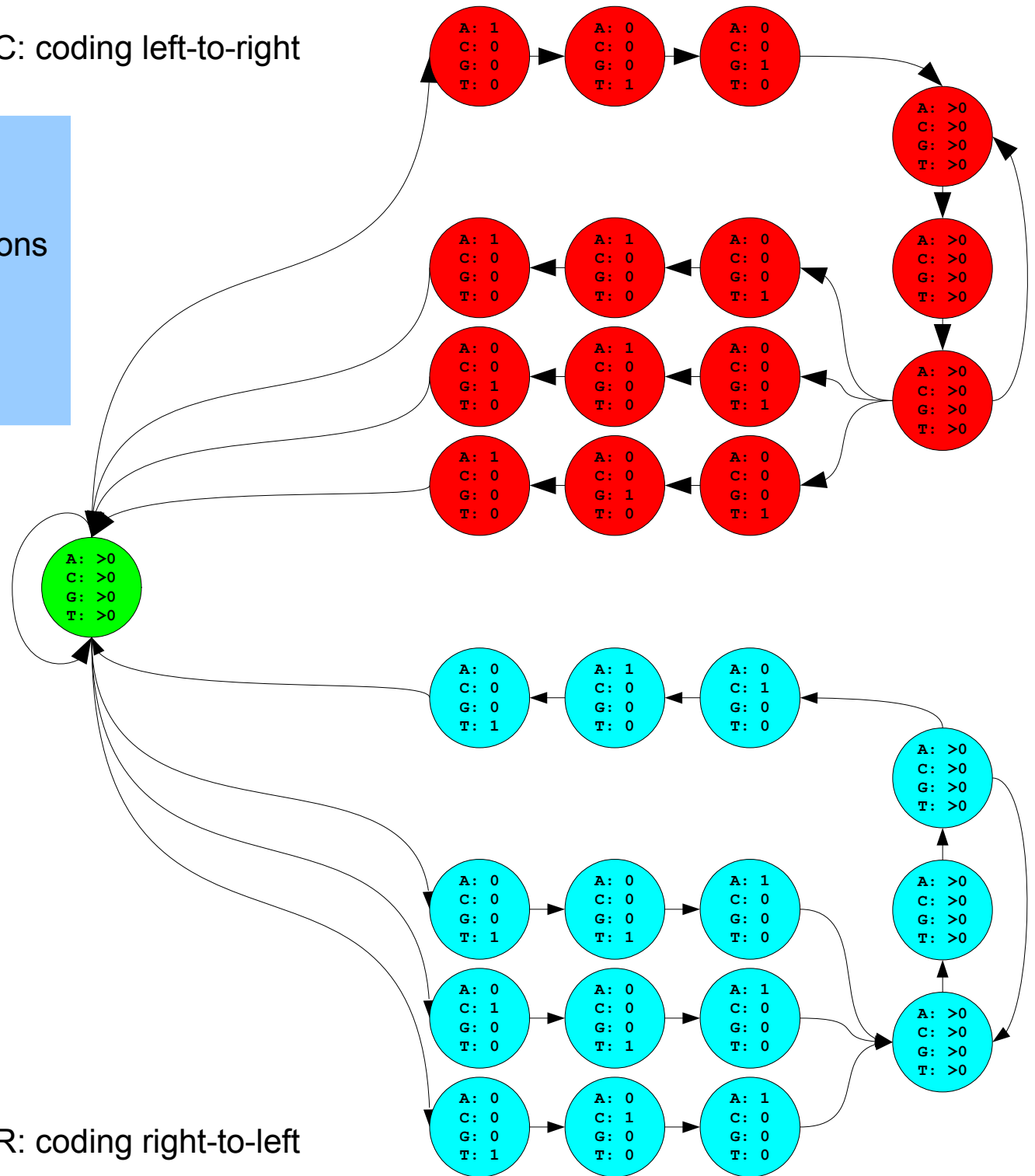
N: Non-coding

C: coding left-to-right

R: coding right-to-left

$$\pi_N = 1$$

$$\pi_C = 0$$



# Analysis of some genomes

```
Length of genome1: 1852441 (1852441)
Length of genome2: 2211485 (2211485)
Length of genome3: 2499279 (2499279)
Length of genome4: 1796846 (1796846)
Length of genome5: 2685015 (2685015)
Length of genome6: 2127839 (2127839)
Length of genome7: 2742531 (2742531)
Length of genome8: 2046115 (2046115)
Length of genome9: 2388435 (2388435)
Length of genome10: 1570485 (1570485)
Length of genome11: 2096309 (2096309)
```

Start-codon in normal genes:

```
ATG [8423, 'NCCC']
ATC [3, 'NCCC']
ATA [1, 'RCCC']
GTG [713, 'NCCC']
ATT [3, 'NCCC']
CTG [2, 'NCCC']
GTT [1, 'NCCC']
CTC [1, 'NCCC']
TTA [1, 'NCCC']
TTG [1020, 'NCCC']
```

Stop-codon in normal genes:

```
TAG [1949, 'CCCN']
TGA [1531, 'CCCN']
TAA [6686, 'CCCN']
```

Reversed stop-codon in reversed genes:

```
TTA (reverse-complement: TAA) [6596, 'NRRR']
CTA (reverse-complement: TAG) [2014, 'NRRR']
TCA (reverse-complement: TGA) [1148, 'NRRR']
```

Reversed start-codon in reversed genes:

```
TAT (reverse-complement: ATA) [2, 'RRRN']
ATG (reverse-complement: CAT) [1, 'RRRN']
GAT (reverse-complement: ATC) [1, 'RRRN']
CAT (reverse-complement: ATG) [8077, 'RRRN']
AAT (reverse-complement: ATT) [4, 'RRRN']
TAC (reverse-complement: GTA) [1, 'RRRN']
CAC (reverse-complement: GTG) [715, 'RRRN']
CAA (reverse-complement: TTG) [953, 'RRRN']
CAG (reverse-complement: CTG) [4, 'RRRN']
```