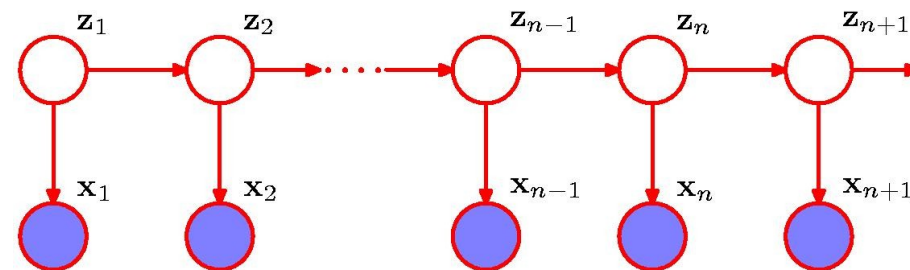


Hidden Markov Models

Selecting the initial model parameters

Using HMMs for (simple) gene finding



Christian Nørgaard Storm Pedersen

cstorm@birc.au.dk

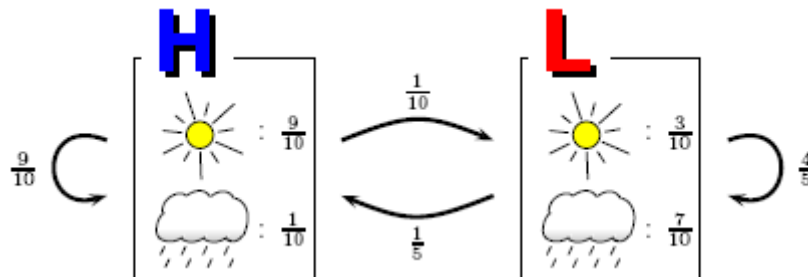
Last time

Training, or how to select model parameters (transition and emission probabilities) to reflect either a set of corresponding $(\mathbf{X}, \mathbf{Z})$'s (Training by Counting), or just a set of $\mathbf{X}$'s (Viterbi Training, and EM Training).

HMMs as a generative model

A HMM *generates a sequence of observables* by moving from hidden state to hidden state according to the transition probabilities and *emitting an observable* (from a discrete set of observables, i.e. a finite alphabet) from each hidden state visited *according to the emission probabilities* of the state ...

Model M :



A *run* follows a sequence of states:

H H L L H

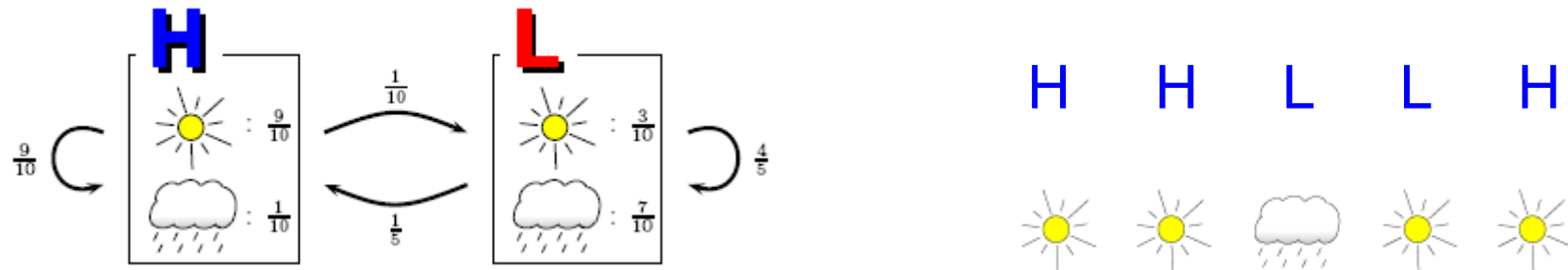
And *emits* a sequence of symbols:



For a HMM that generates finite strings (e.g. a HMM with an end-state), the language $L = \{\mathbf{X} \mid p(\mathbf{X}) > 0\}$ is regular ...

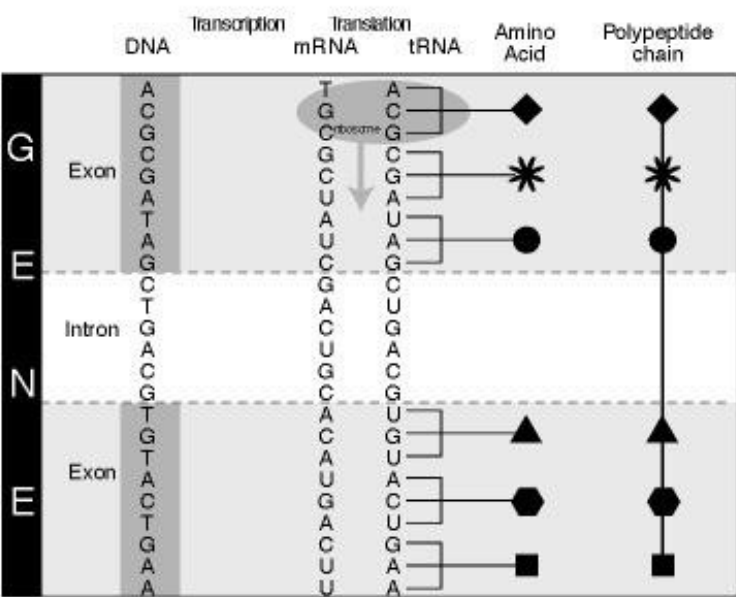
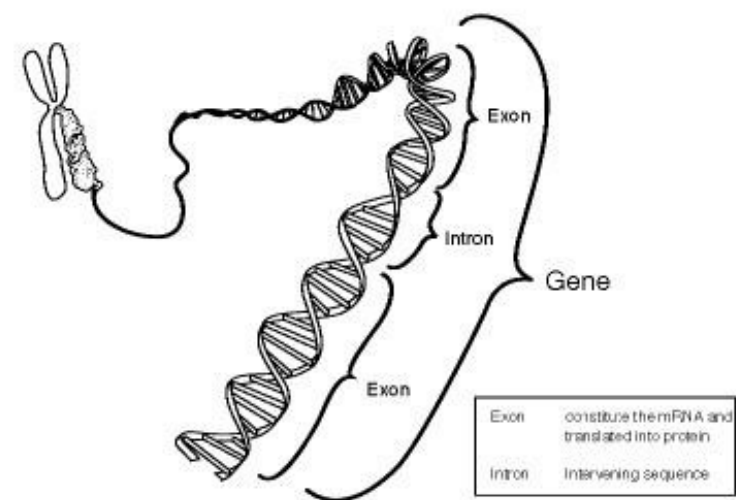
Selecting initial model parameters

The initial selection of transition and emission probabilities, i.e. A , π , Φ , models possible explanations (or the underlying structure) of possible observations, i.e. the “syntax” of possible sequences of observations, recall that the language $L = \{x \mid P(x \mid \theta) > 0\}$ is regular.



The initial selection of parameters is essential just to decide which parameters are 0 (or 1), i.e. to decide which transitions or emissions should never (or always) be possible ...

Example – Gene finding



Each protein is encoded in a stretch of DNA. A **gene** ...

Which is **expressed** when the protein is needed ...

Important problem

Locating genes on the genome and determining how they get expressed ...

Recognizing the patterns that indicates a gene ...

GENETIC CODE CRACKED FULL STORY

2ND
1ST

U	C	A	G
PHE PHE LEU LEU	SER SER SER SER	TYR TYR Ochre Amber	CYS CYS Opal TRP
LEU LEU LEU LEU	PRO PRO PRO PRO	HIS HIS GLUN GLUN	ARG ARG ARG ARG
ILEU ILEU ILEU MET	THR THR THR THR	ASPN ASPN LYS LYS	SER SER ARG ARG
VAL VAL VAL VAL	ALA ALA ALA ALA	ASP ASP GLU GLU	GLY GLY GLY GLY

3RD
KEY

PHE - PHENYLALANINE
GLU - GLUTAMIC ACID
ASP - ASPARTIC ACID
ASPN - ASPARAGINE
ILEU - ISOLEUCINE
MET - METHIONINE
THR - THREONINE
ARG - ARGinine
GLUN - GLUTAMINE
HIS - HISTIDINE
TRP - TRYPTOPHAN
TYR - TYROSINE
CYS - CYSTEINE
LEU - LEUCINE
PRO - PROLINE
ALA - ALANINE
VAL - VALINE
GLY - GLYCINE
LYS - LYSINE
SER - SERINE

DEFENSE
RAT
AT
SF

Here it is. The code for each of the twenty amino acids. So simple isn't it? Read the table and you can't miss it.

>NC_002737.1 Streptococcus pyogenes M1 GAS
TTGTTGATATTCTGTTTTTCTTTTTTAGTTTTCCACATGAAAAATAGTTGAAAACAATA
GCGGTGTCCCCTTAAAATGGCTTTTCCACAGGTTGTGGAGAACCCAAATTAACAGTGTTA
ATTTATTTTCCACAGGTTGTGGAAAACTAACTATTATCCATCGTTCTGTGGAAAACTAG
AATAGTTTATGGTAGAATAGTTCTAGAATTATCCACAAGAAGGAACCTAGTATGACTGAA
AATGAACAAATTTTTTGGAACAGGGTCTTGGAATTAGCTCAGAGTCAATTAACAGGCA
ACTTATGAATTTTTTGTTCATGATGCCCCTCTATTAAAGGTCGATAAGCATATTGCAACT
ATTTACTTAGATCAAATGAAAGAGCTCTTTTGGGAAAAAAATCTTAAAGATGTTATTCTT
ACTGCTGGTTTTGAAGTTTATAACGCTCAAATTTCTGTTGACTATGTTTTCGAAGAAGAC
CTAATGATTGAGCAAAATCAGACCAAAATCAACCAAAAACCTAAGCAGCAAGCCTTAAAT
TCTTTGCCTACTGTTACTTCAGATTTAACTCGAAATATAGTTTTGAAAACTTTATTCAA
GGAGATGAAAATCGTTGGGCTGTTGCTGCTTCAATAGCAGTAGCTAATACTCCTGGA
ACTACCTATAATCCTTTGTTTATTTGGGGTGGCCCTGGGCTTGGAAAAACCCATTTATTA
AATGCTATTGGTAATTCTGTACTATTAGAAAATCCAAATGCTCGAATTAAATATATCACAG
CTGAAAACTTTATTAATGAGTTTGTTATCCATATTCGCCTTGATACCATGGATGAATTGAA
GAAAAATTTTCGAATTTAGATTTACTCCTTATTGATGATATCCAATCTTTAGCTAAAAAA
ACGCTCTCTGGAACACAAGAAGAGTTCTTTAATACTTTTAAATGCACTTCATAATAATAAC
AAACAAATTGTCCTAACAAGCGACCGTACACCAGATCATCTCAATGATTTAGAAGATCGA
TTAGTTACTCGTTTTAAATGGGGATTAAACAGTCAATATCACACCTCCTGATTTTGAAACA
CGAGTGGCTATTTTGACAAATAAAATTCAAGAATATAACTTTATTTTTCCTCAAGATACC
ATTGAGTATTTGGCTGGTCAATTTGATTCTAATGTCAGAGATTTAGAAGGTGCCTTAAAA
GATATTAGTCTGGTTGCTAATTTCAAACAAATTGACACGATTACTGTTGACATTGCTGCC
GAAGCTATTCGCGCCAGAAAGCAAGATGGACCTAAAATGACAGTTATTTCCATCGAAGAA
ATTCAAGCGCAAGTTGGAAAAATTTACGGTGTTACCGTCAAAGAAATTAAGCTACTAAA
CGAACACAAAATATTGTTTTAGCAAGACAAGTAGCTATGTTTTTAGCACGTGAAATGACA
GATAACAGTCTTCCTAAAATTTGAAAAAGAATTTGGTGGCAGAGACCATTAACAGTACTC
CATGCCTATAATAAAATCAAAAACATGATCAGCCAGGACGAAAGCCTTAGGATCGAAATT
GAAACCATAAAAAACAAAATTAATAACATGTGAAAAAGAATATCTTTTATGAAATAGTT
ATCCACAAGTTGTGAACATCCATTTAGTCTTGGATTCTCTCGTTTATTTAGAGTTATCCA
CTATATACACAAGACCTACTACTACTATTATTATACTTATTAAATAAAGGAGTTCT

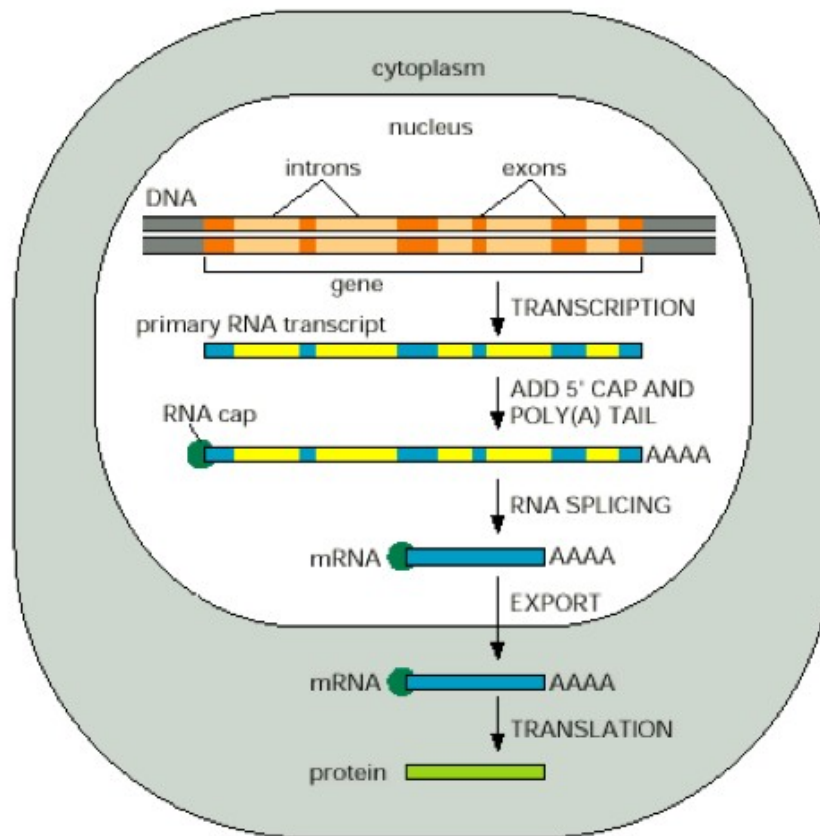
[illegible]

Design a HMM that models the syntax of genes

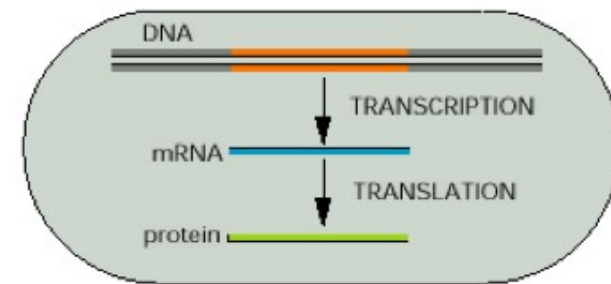
Gene structure

Depends on the organism (eucaryote or procaryote)

(A) EUCARYOTES



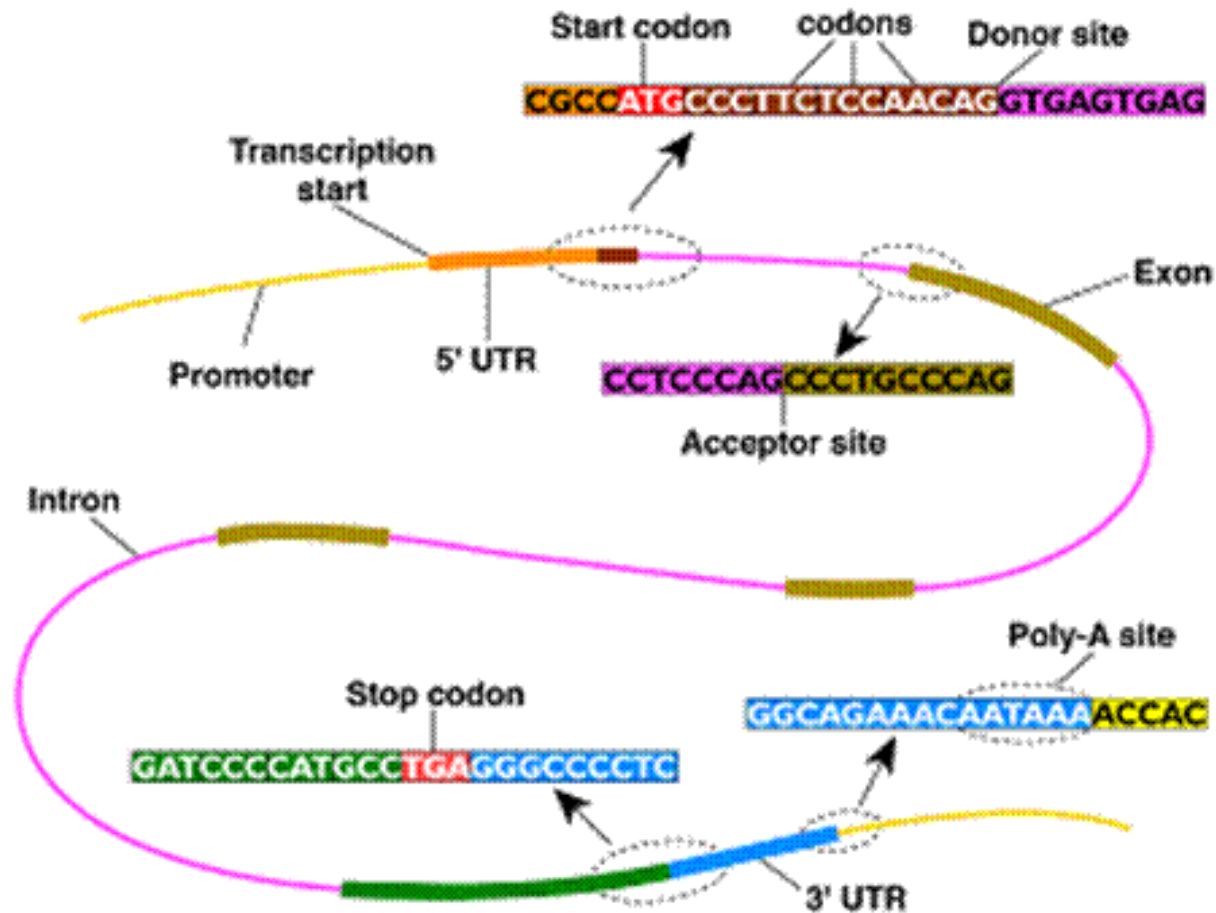
(B) PROCARYOTES



Smaller genomes and high coding density.

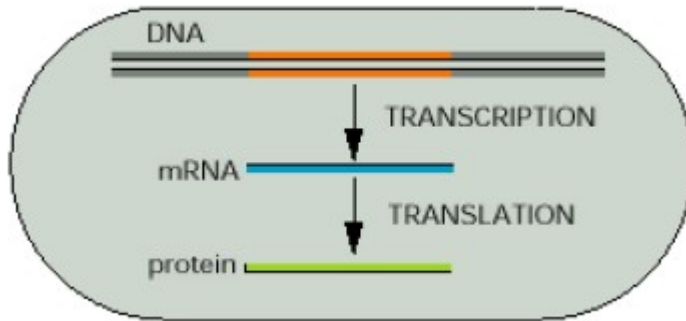
Large genomes. Intron/exon structure and low coding density

Gene structure in eukaryotes



Eukaryotic gene structure in more details

Gene structure in procaryotes

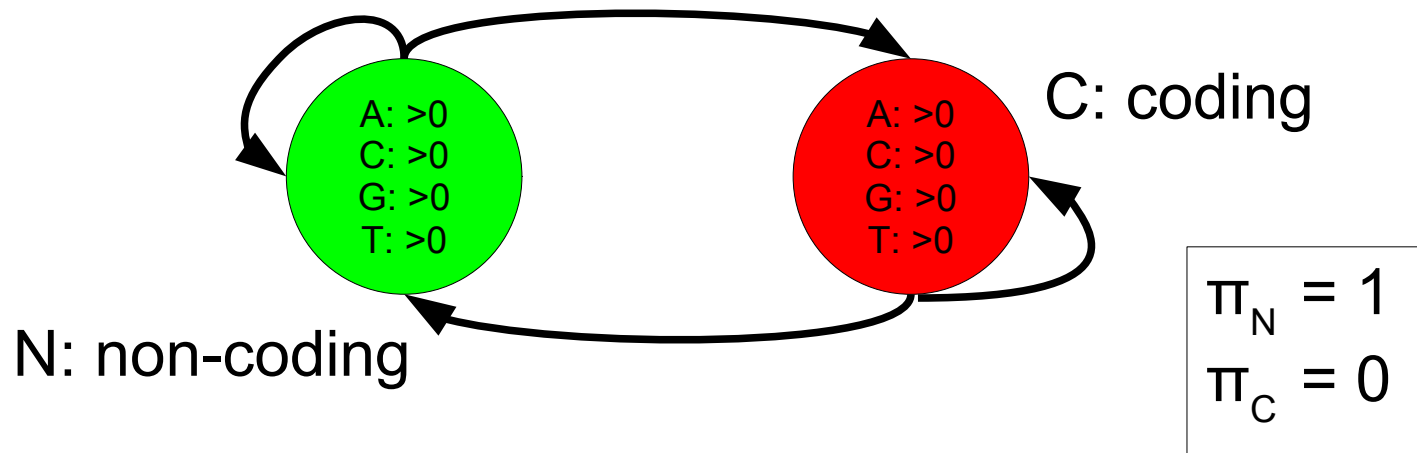


Biological facts

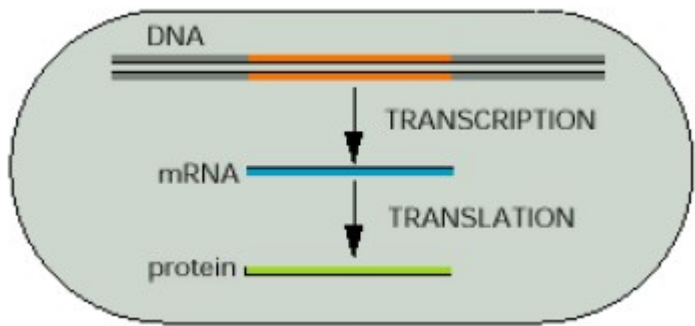
- The gene is a substring of the DNA sequence of A,C,G,T's

Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacatgcag



Gene structure in procaryotes

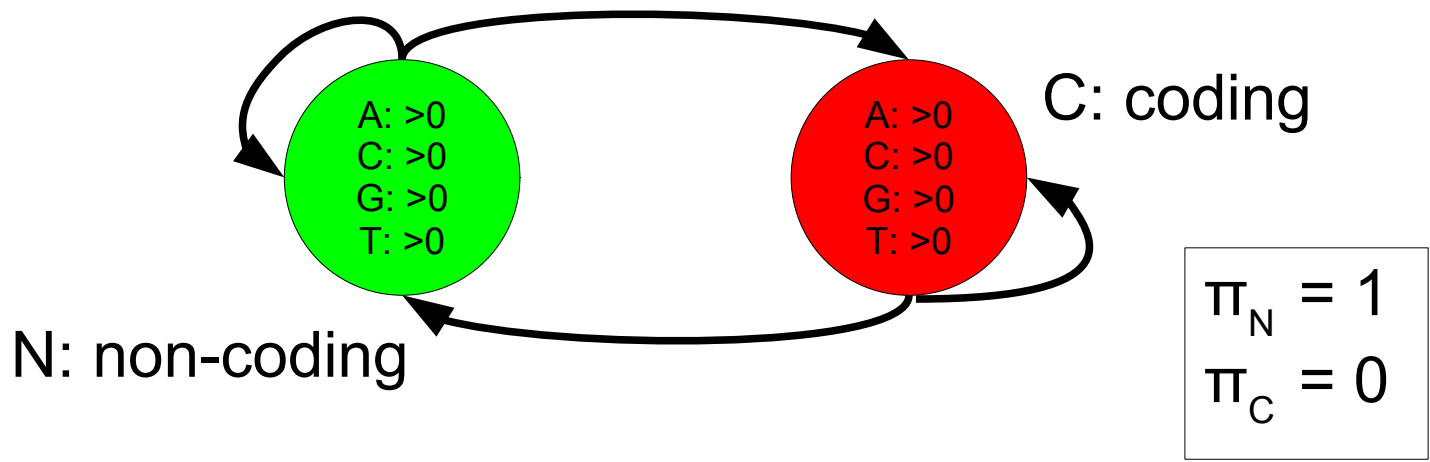


Biological facts

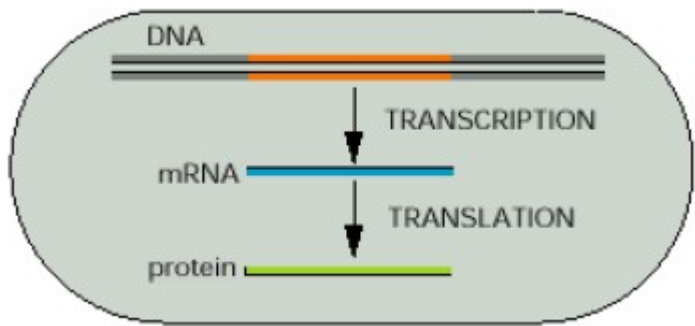
- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**

Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacatgcag



Gene structure in procaryotes

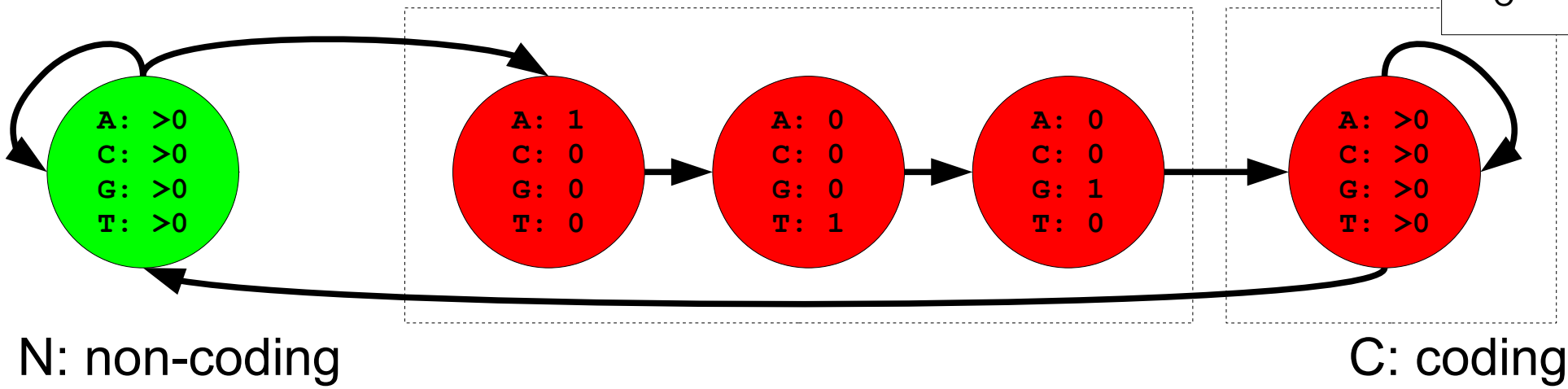


- Biological facts**
- The gene is a substring of the DNA sequence of A,C,G,T's
 - The gene starts with a start-codon **atg**

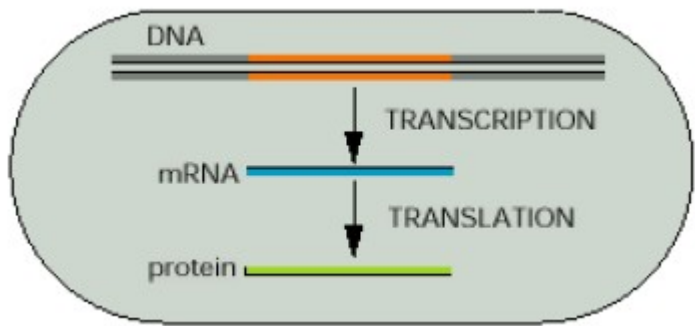
Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$\pi_N = 1$
 $\pi_C = 0$



Gene structure in procaryotes

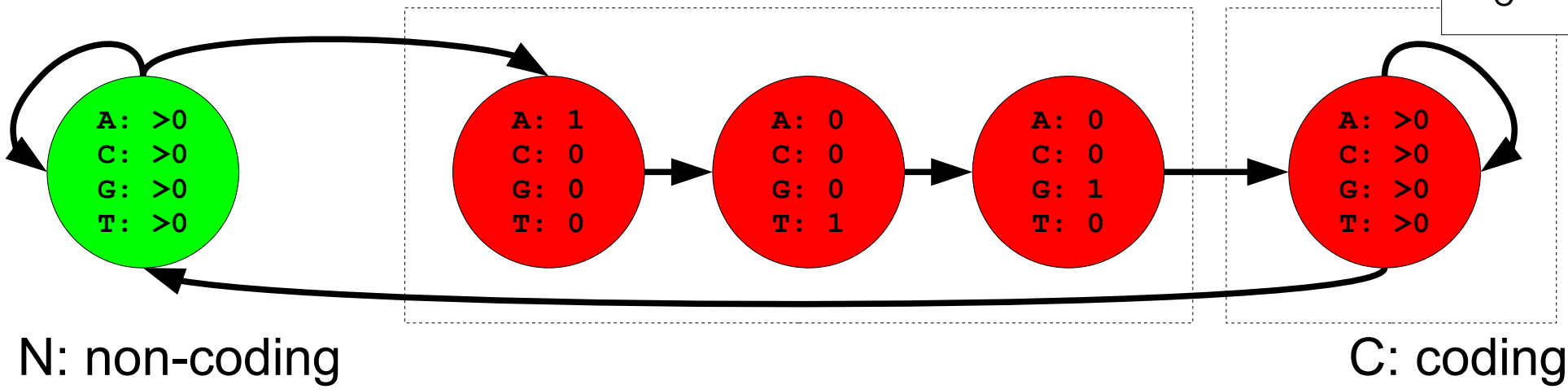


- Biological facts**
- The gene is a substring of the DNA sequence of A,C,G,T's
 - The gene starts with a start-codon **atg**
 - The gene ends with a stop-codon **taa**, **tag** or **tga**

Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$\pi_N = 1$
 $\pi_C = 0$

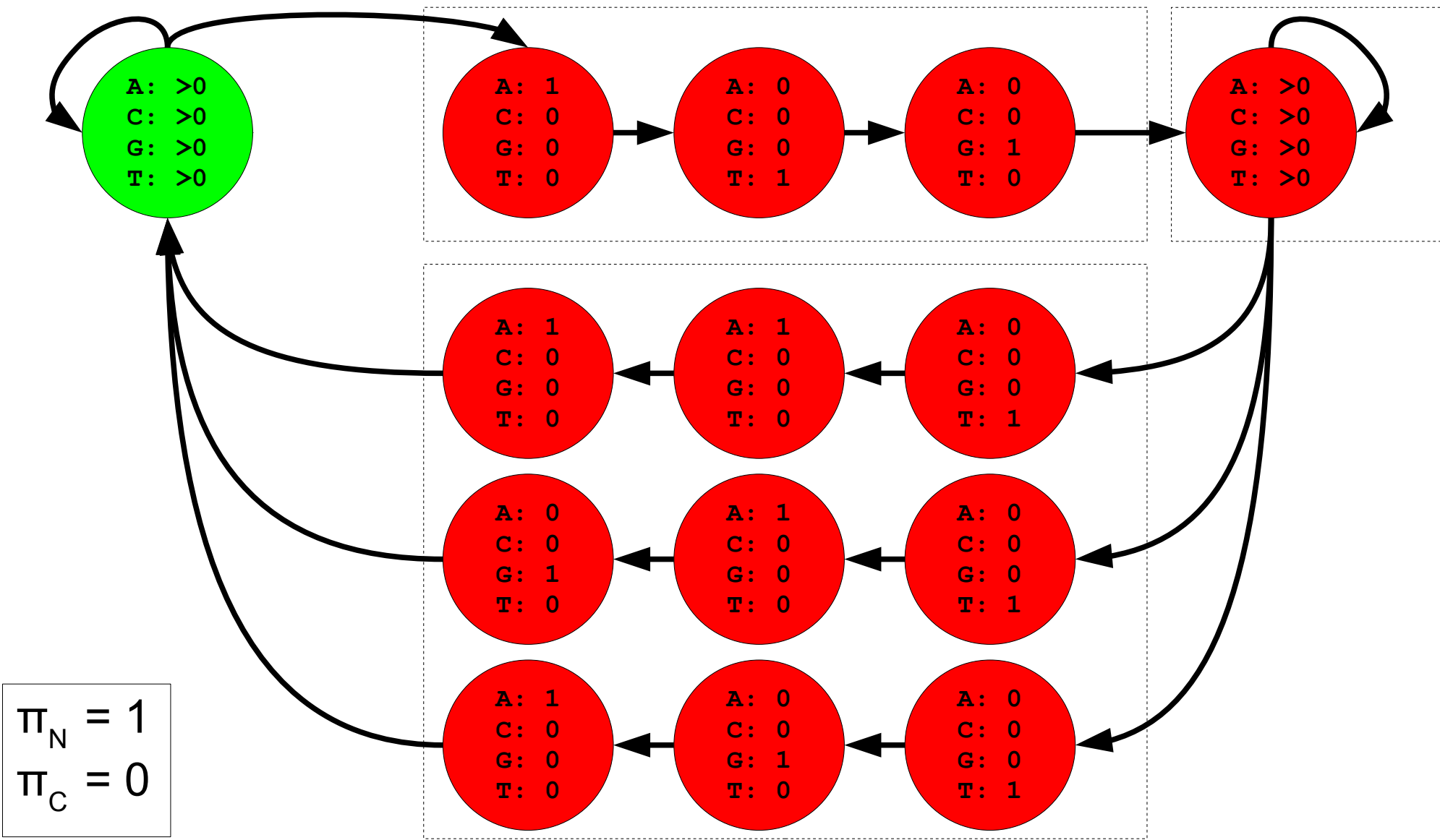


Gene structure

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**

N: non-coding

C: coding

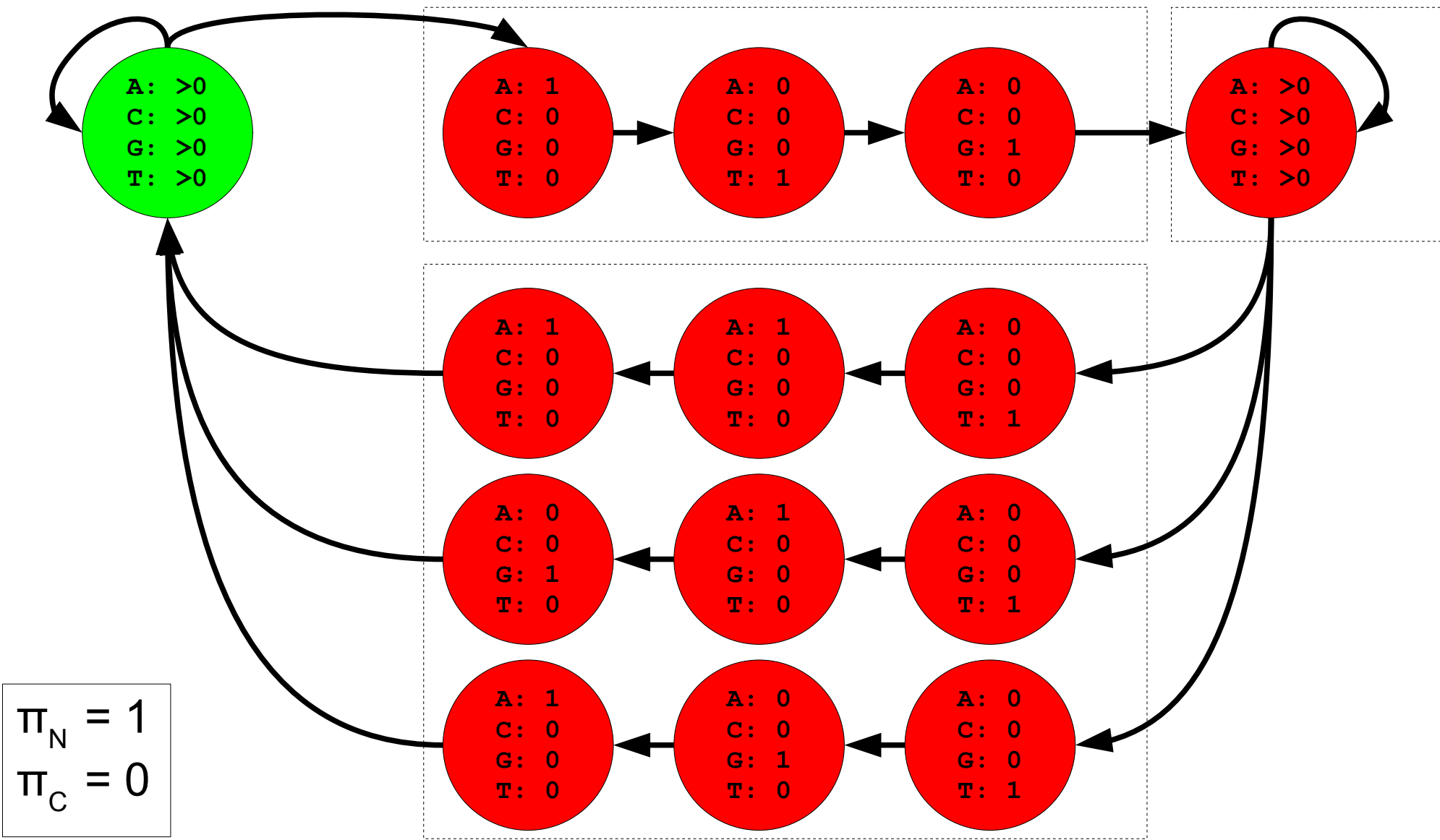


Gene structure

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**
- The number of nucleotides in a gene is a multiplum of 3

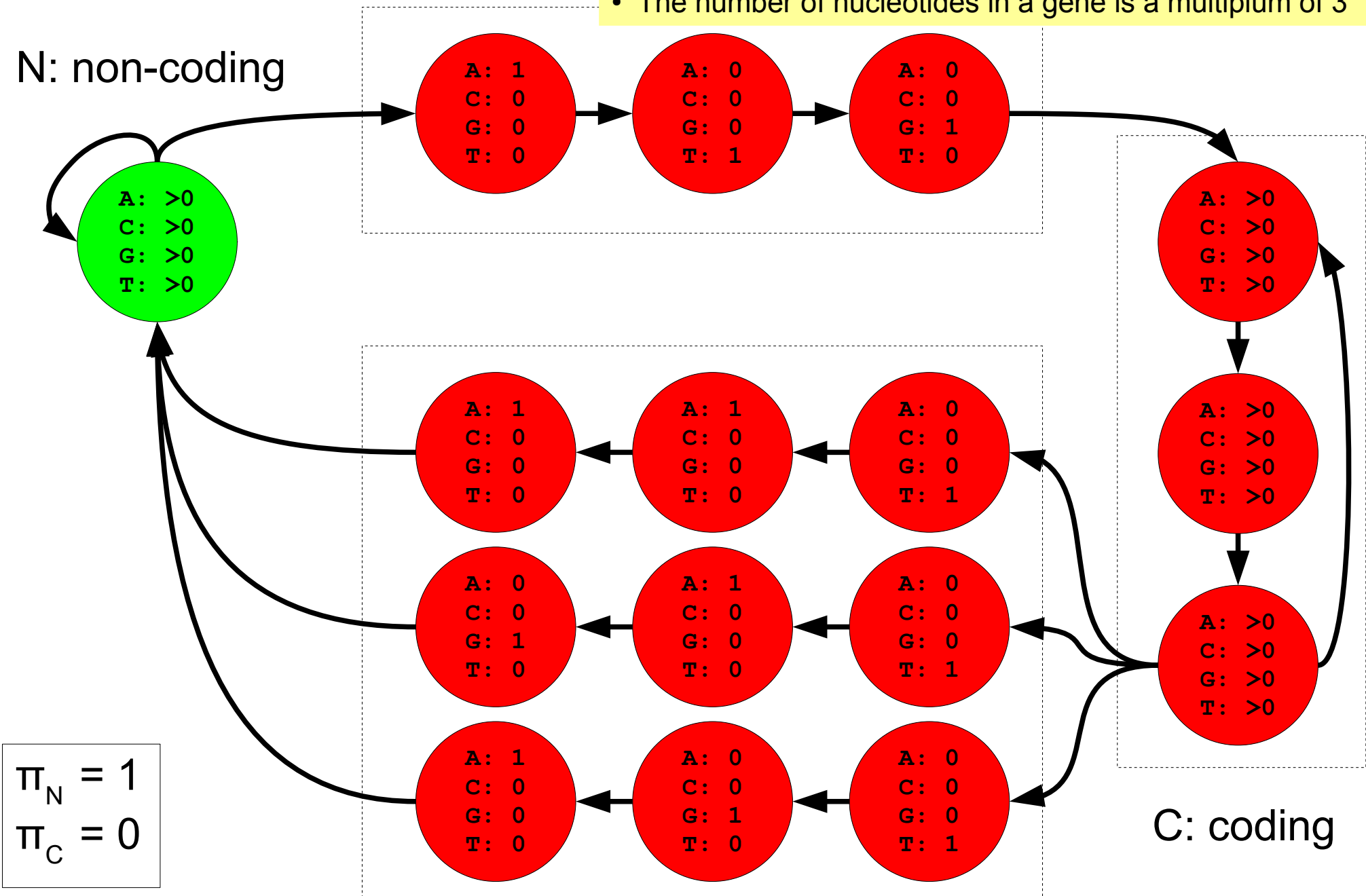
N: non-coding

C: coding

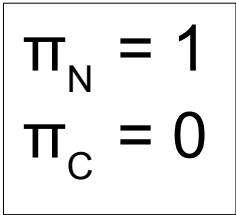


Gene structure

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**
- The gene ends with a stop-codon **taa**, **tag** or **tga**
- The number of nucleotides in a gene is a multiplum of 3

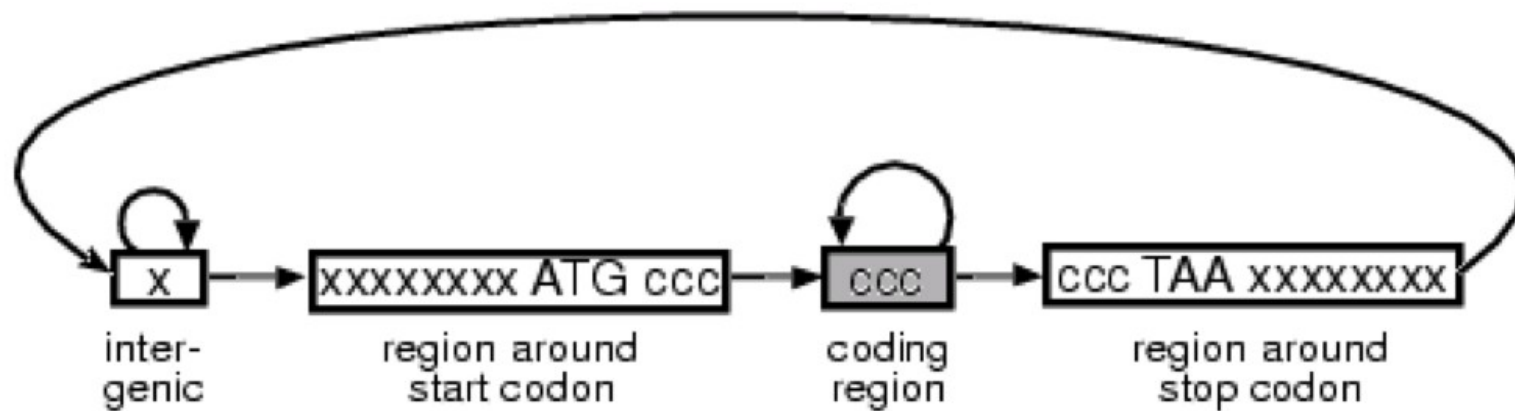


N: non-coding



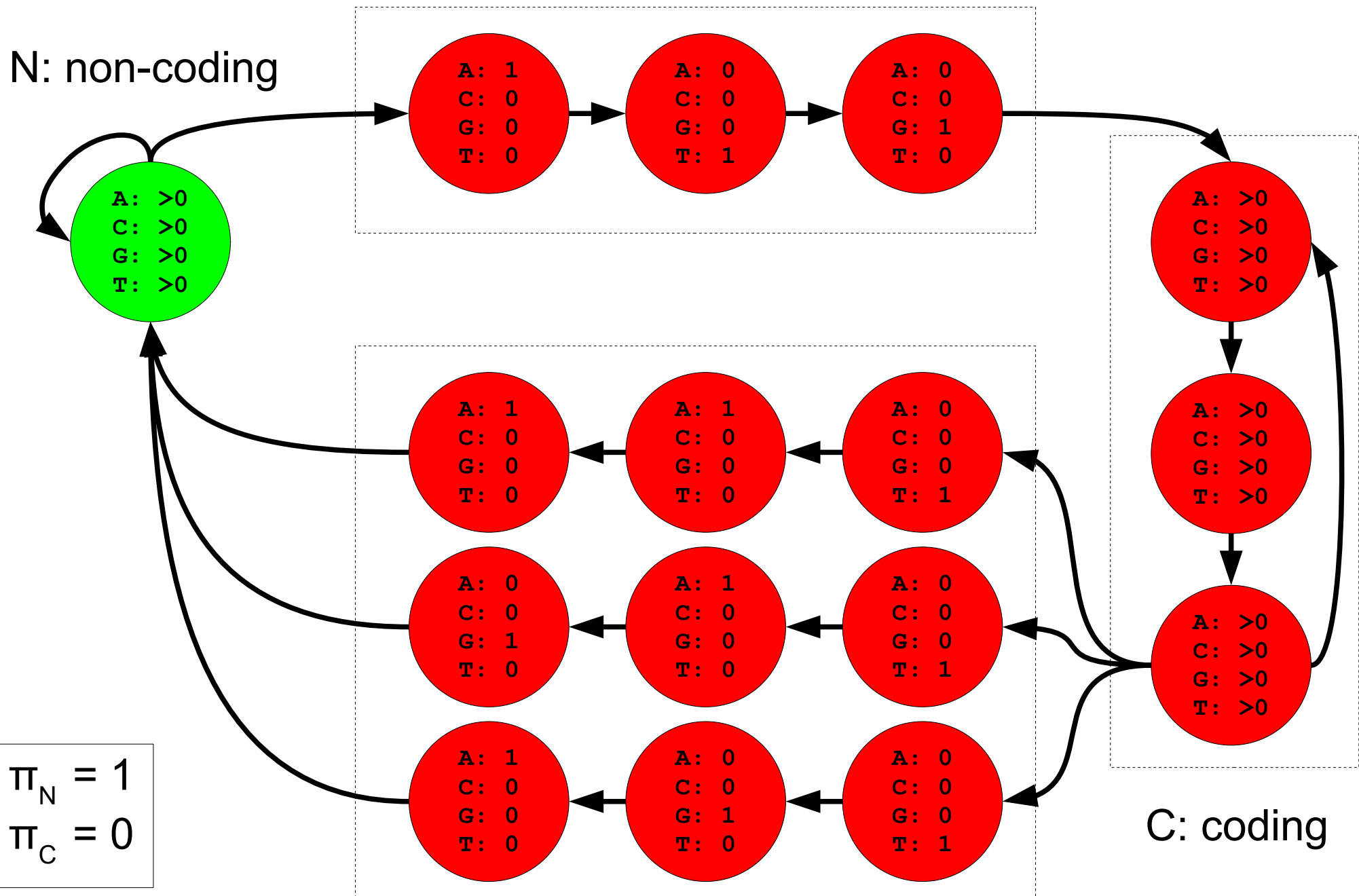
C: coding

Gene structure in procaryotes



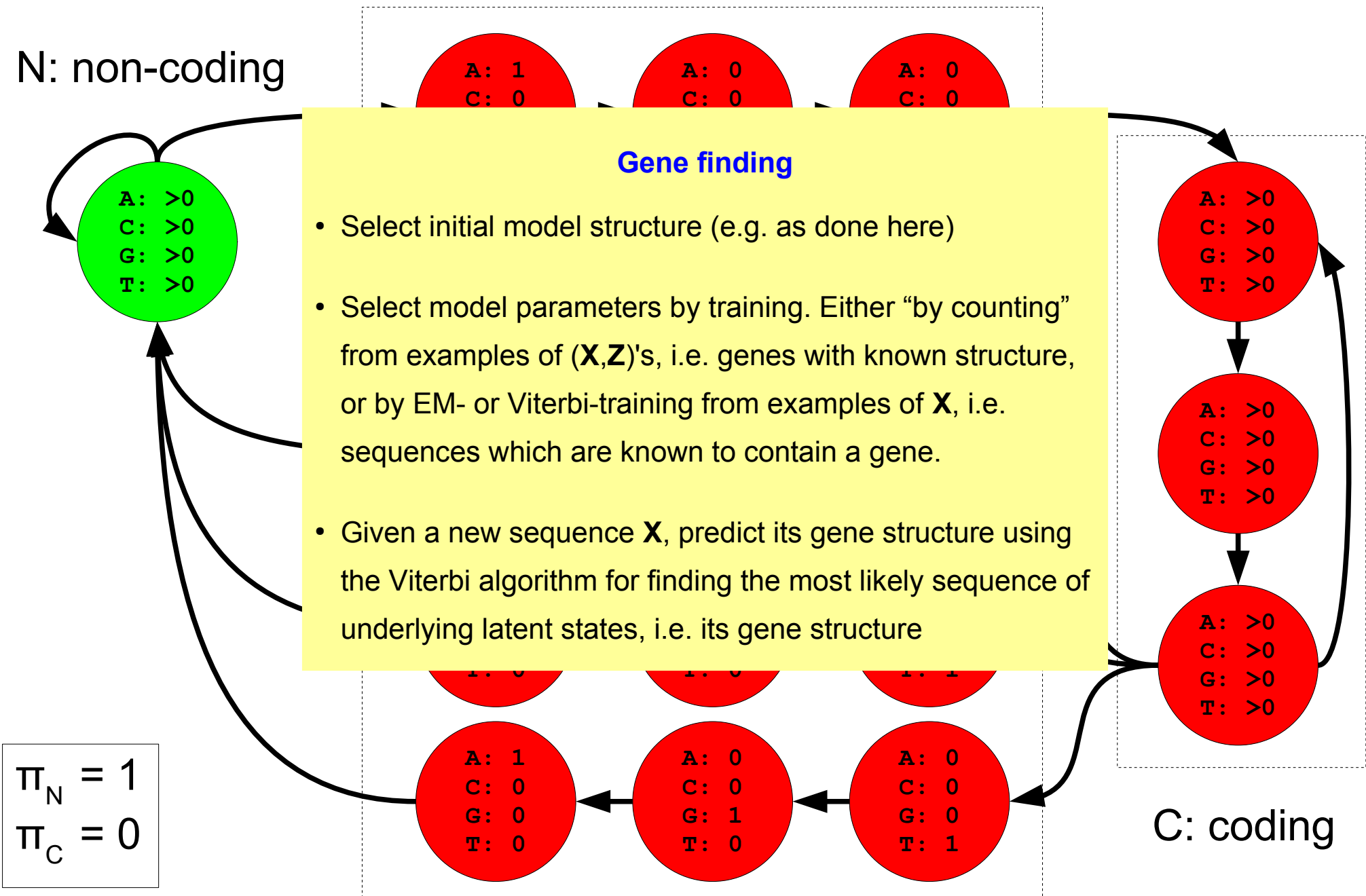
From “An Introduction to HMMs for Biological Sequences”, A. Krogh, 1998

Example – Gene finding



Example – Gene finding

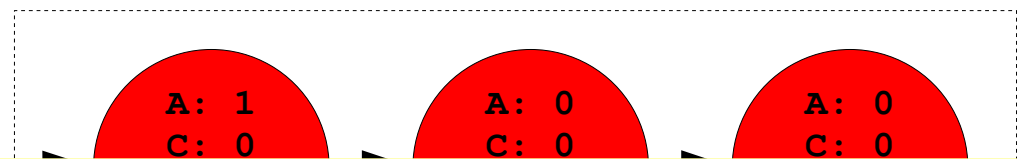
N: non-coding



C: coding

Example – Gene finding

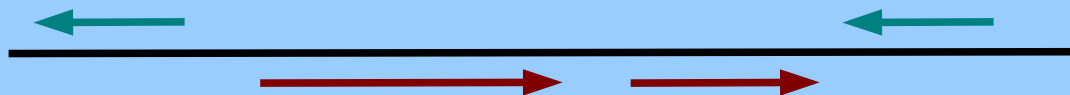
N: non-coding

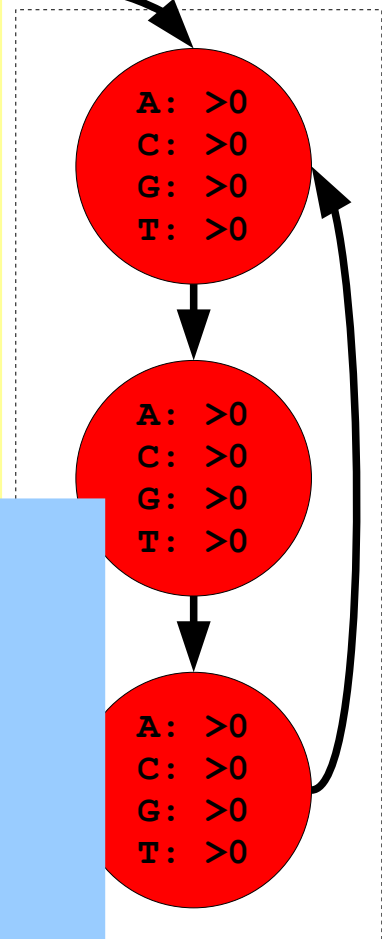


Gene finding

- Select initial model structure (e.g. as done here)
- Select model parameters by training. Either “by counting” from examples of $(\mathbf{X}, \mathbf{Z})$'s, i.e. genes with known structure, or by EM- or Viterbi-training from examples of $\mathbf{X}$, i.e. sequences which are known to contain a gene.

Even more biology

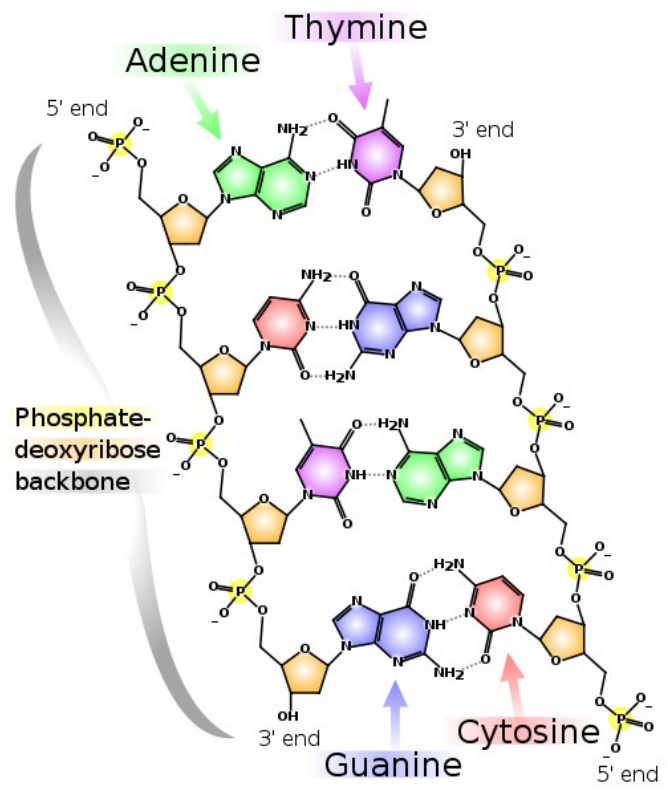
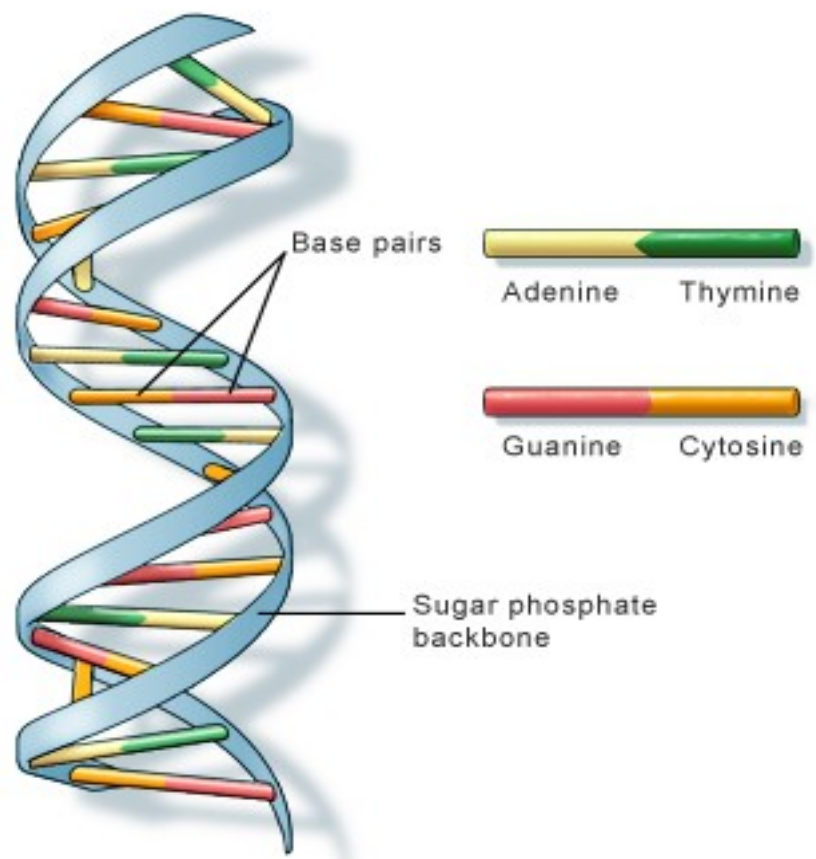
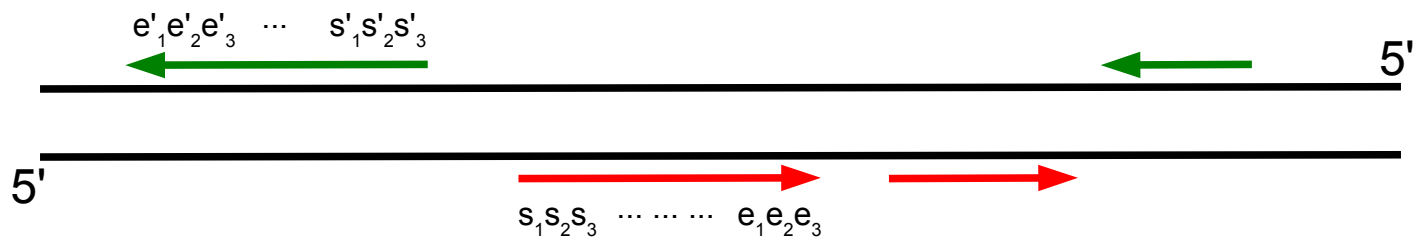
- There can be genes in both directions (and over lapping)

- There are more possible start-codons **atg**, **gtg**, and **ttg**
- Internal codons cannot be stop-codons
- And a lot more ...



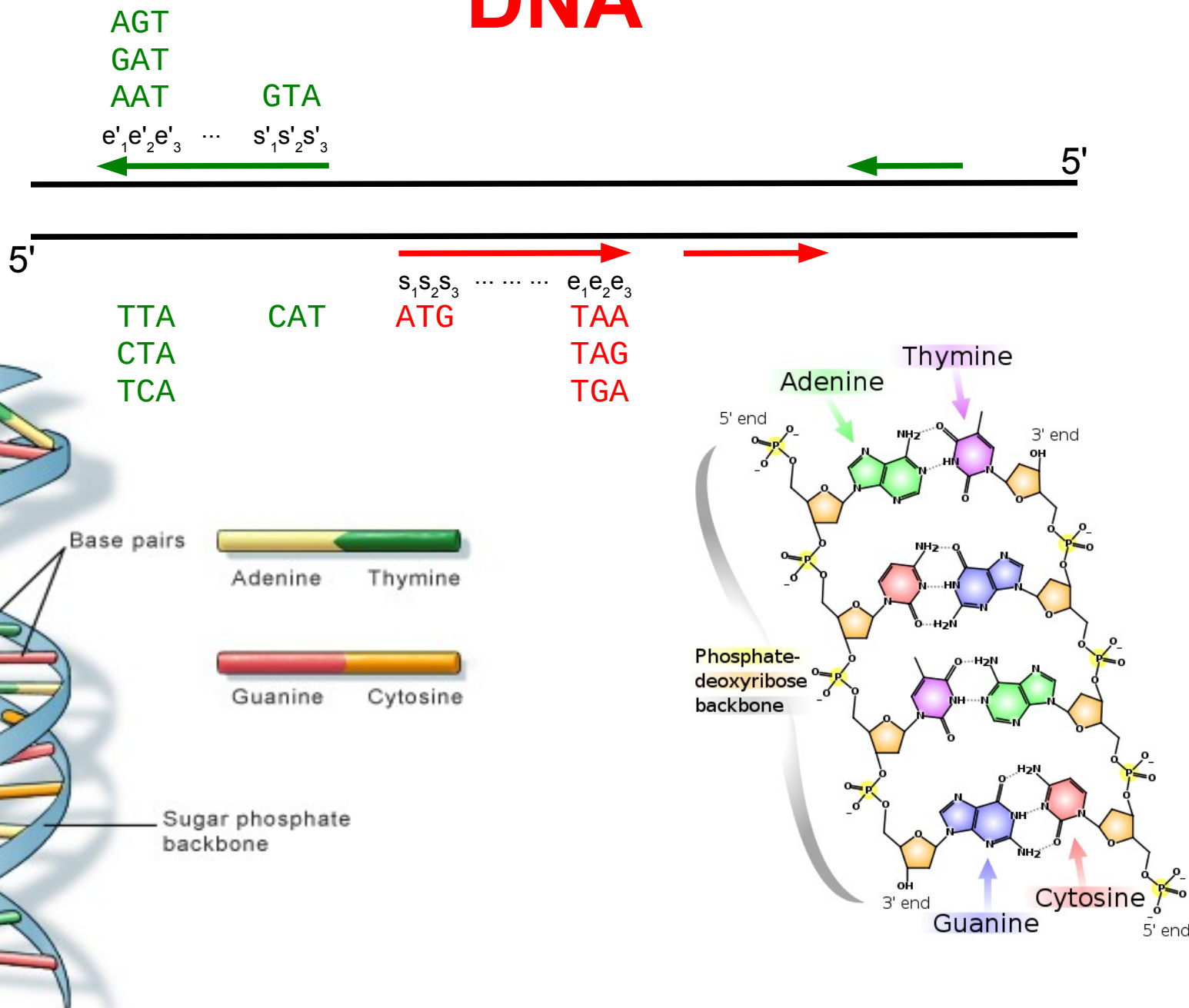
C: coding

$$\pi_N = 1$$
$$\pi_C = 0$$

DNA



DNA



C: coding left-to-right

Even more biology

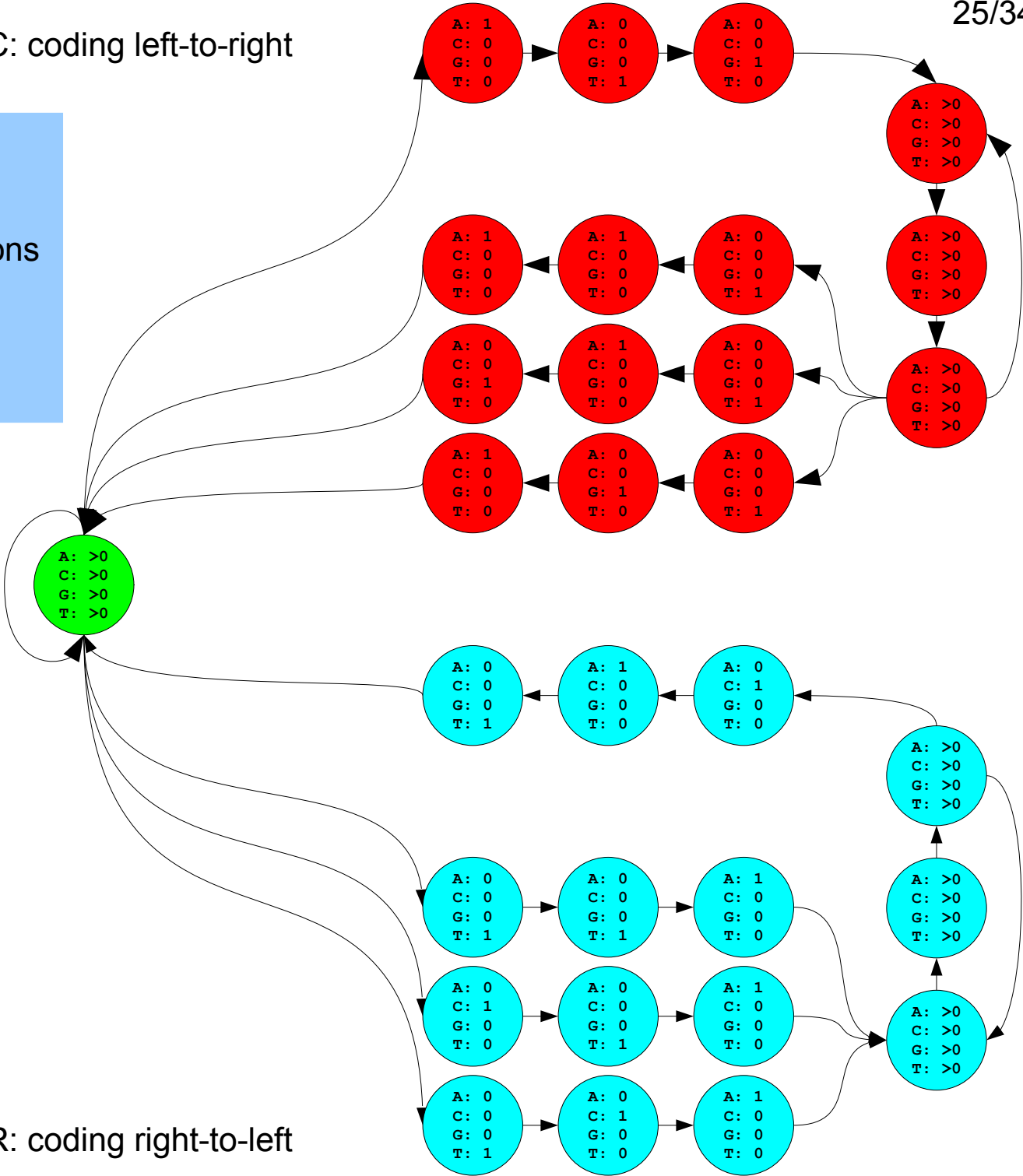
There can be genes in both directions



N: Non-coding

$$\pi_N = 1$$
$$\pi_C = 0$$

R: coding right-to-left



Example – 7-state HMM

Observable: {A, C, G, T}, States: {0,1, 2, 3, 4, 5, 6}

A

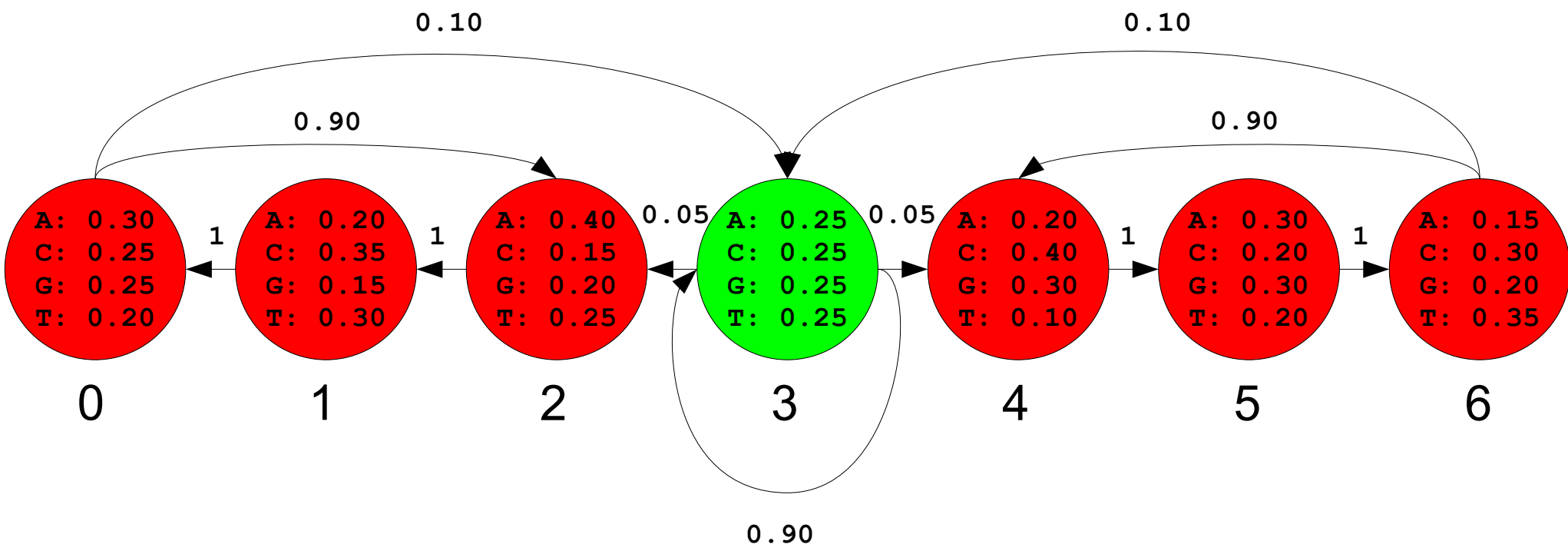
0.00	0.00	0.90	0.10	0.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.05	0.90	0.05	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	1.00
0.00	0.00	0.00	0.10	0.90	0.00	0.00

π

0.00
0.00
0.00
1.00
0.00
0.00
0.00

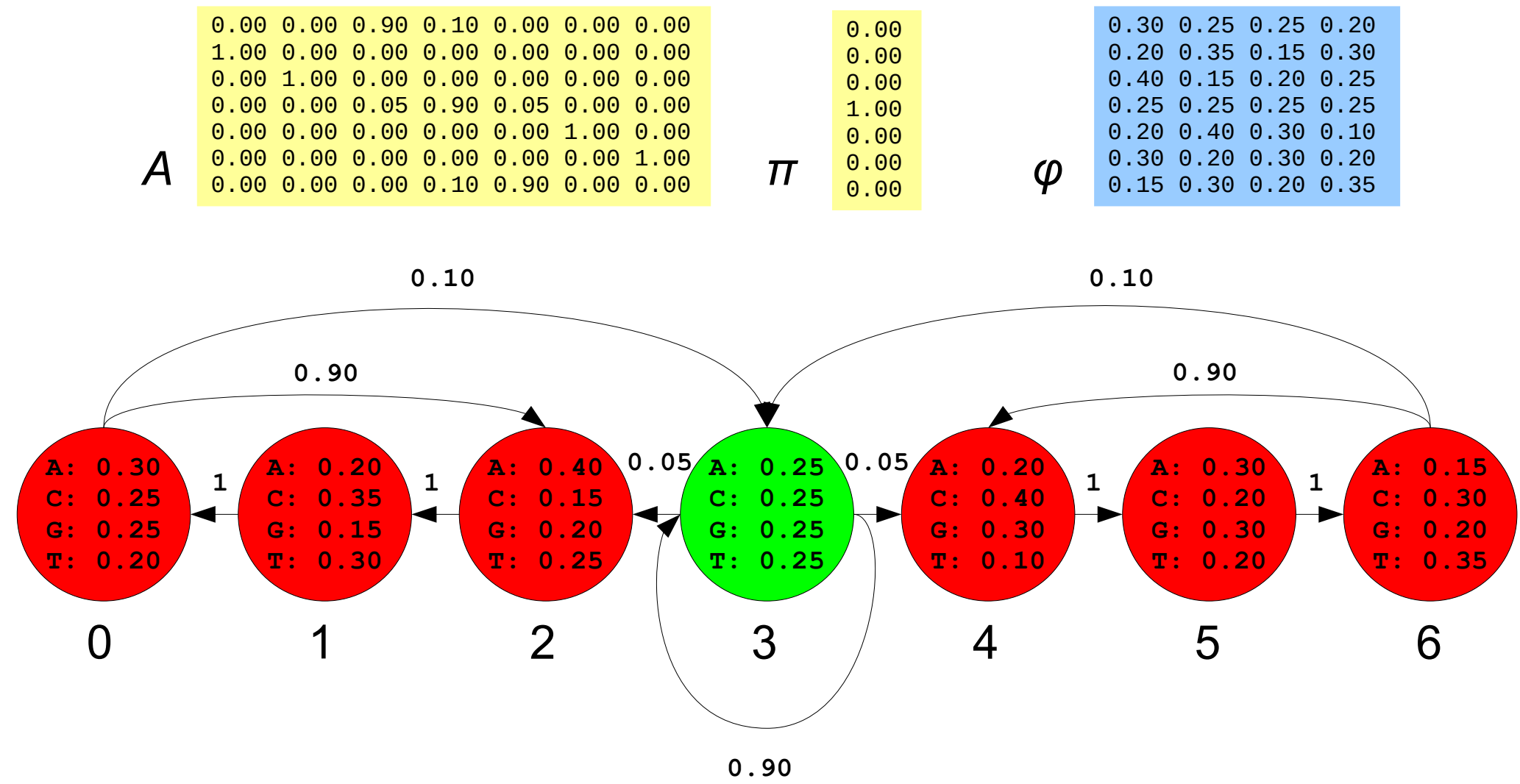
φ

0.30	0.25	0.25	0.20
0.20	0.35	0.15	0.30
0.40	0.15	0.20	0.25
0.25	0.25	0.25	0.25
0.20	0.40	0.30	0.10
0.30	0.20	0.30	0.20
0.15	0.30	0.20	0.35

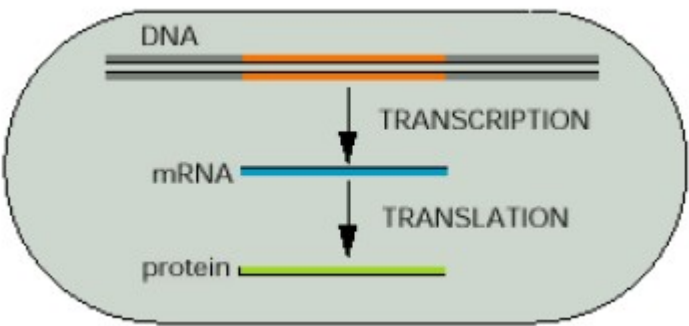


This model is also applicable for gene finding.

It does not model start- and stop-codons explicitly, but models that genes in both directions are a sequence of triplets.



Problem: From annotation to Z



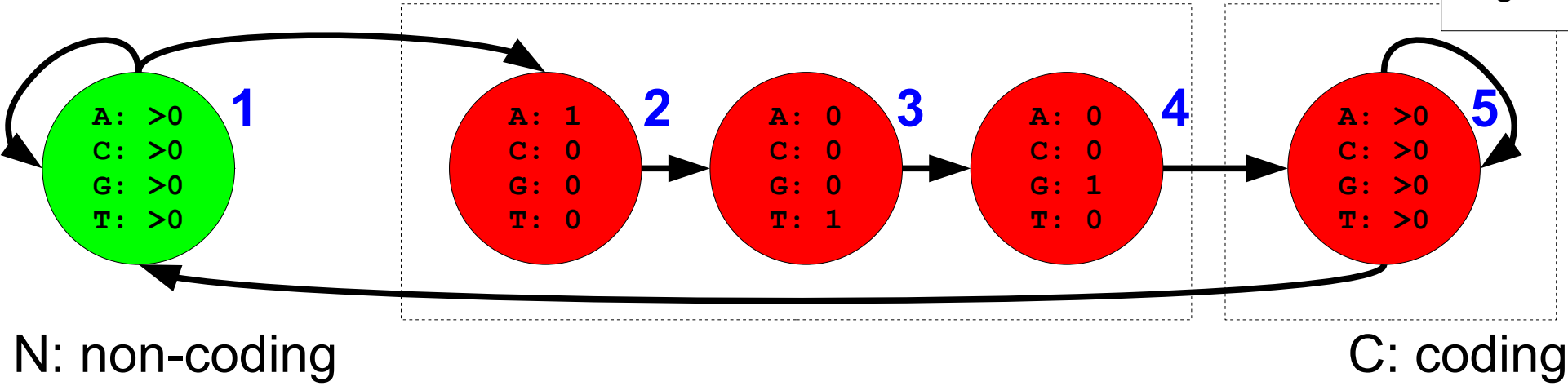
Biological facts

- The gene is a substring of the DNA sequence of A,C,G,T's
- The gene starts with a start-codon **atg**

Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$\pi_N = 1$
 $\pi_C = 0$



Problem: From annotation to Z

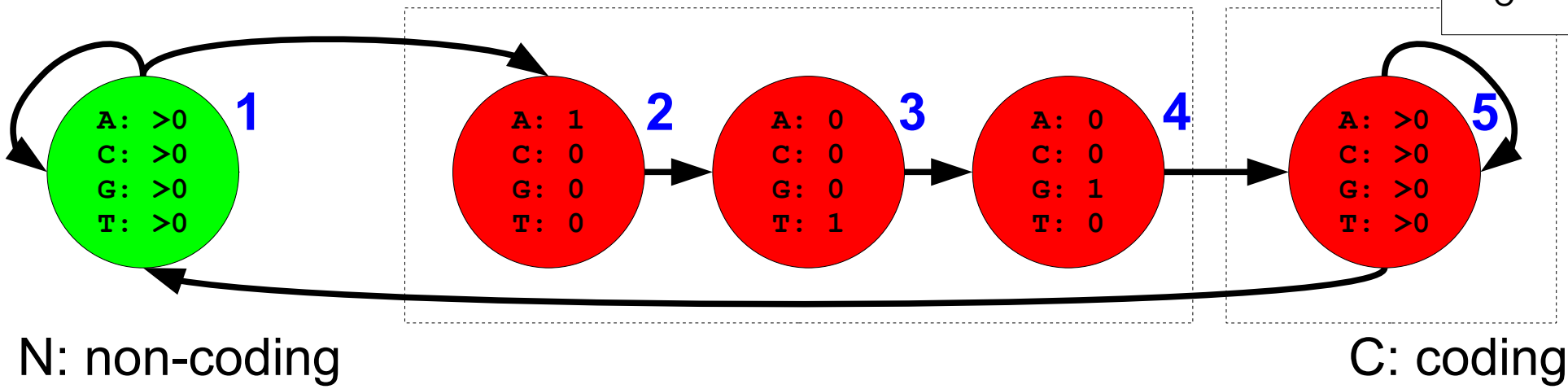
Problem: The string **Z**=NNNCCC.... is not a proper sequence of states in the illustrated HMM, but it can easily be converted into one (because there in this case is a 1-1 matching between a sequence of Ns and Cs and a sequence of states).

ence of A,C,G,T's

Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$
$$\pi_C = 0$$



Problem: From annotation to Z

Problem: The string **Z**=NNNCCC.... is not a proper sequence of states in the illustrated HMM, but it can easily be converted into one (because there in this case is a 1-1 matching between a sequence of Ns and Cs and a sequence of states).

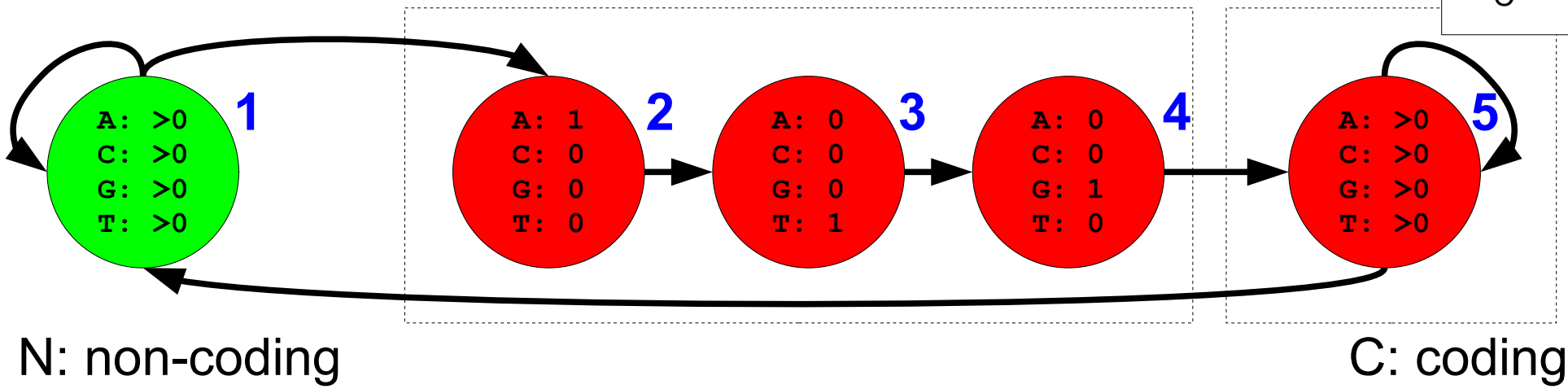
ence of A,C,G,T's

1112345555551111111123455555555555555111111111111

Z: NNNCCCCCCCCNNNNNNNNCCCCCCCCCCCCCCCCNNNNNNNNNNNN

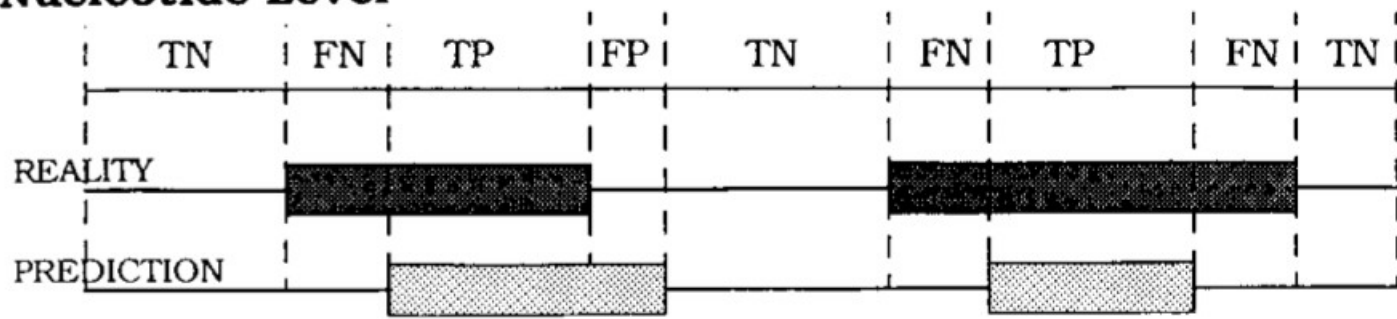
X: acgatgcgctaatatgtccgatgacgtgagcataagcgacat

$$\pi_N = 1$$
$$\pi_C = 0$$



Evaluating performance

Nucleotide Level



		REALITY		
		coding	no coding	
PREDICTION	coding	TP	FP	TP+FP
	no coding	FN	TN	FN+TN
		TP+FN	TN+FP	

$$S_n = \frac{TP}{TP + FN}$$

Sensitivity

$$S_p = \frac{TN}{TN + FP}$$

Specificity

$$CC = \frac{(TP \times TN) - (FN \times FP)}{\sqrt{(TP + FN) \times (TN + FP) \times (TP + FP) \times (TN + FN)}}$$

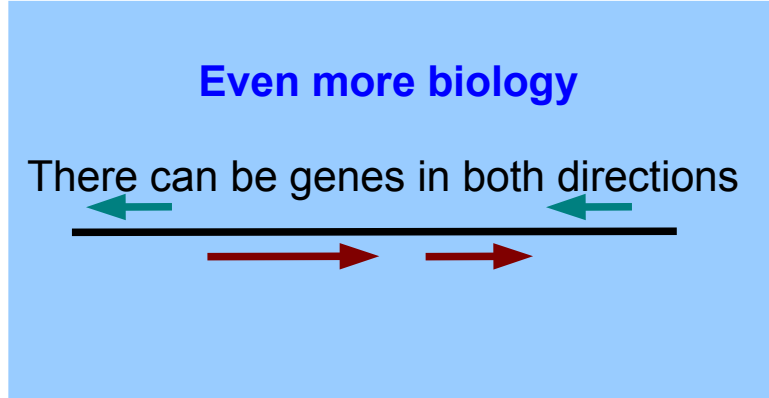
Correlation Coefficient

$$ACP = \frac{1}{4} \left[\frac{TP}{TP + FN} + \frac{TP}{TP + FP} + \frac{TN}{TN + FP} + \frac{TN}{TN + FN} \right]$$

$$AC = (ACP - 0.5) \times 2$$

Approximate Correlation

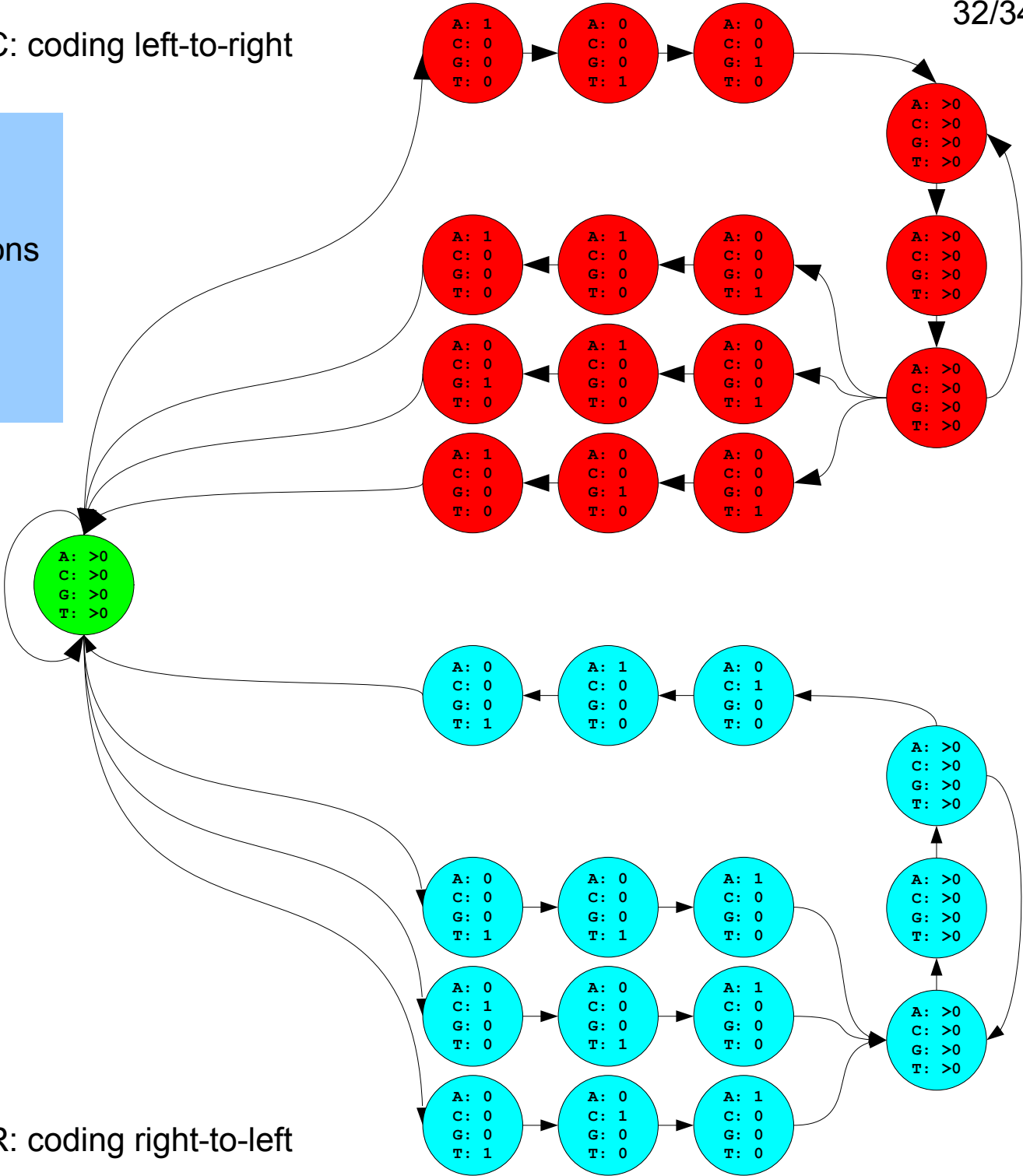
C: coding left-to-right



N: Non-coding

$$\pi_N = 1$$
$$\pi_C = 0$$

R: coding right-to-left



Analysis of some genomes

Even more biology

- There can be genes in both directions
- There are more possible start-codons **atg**, **gtg**, and **ttg**
- Internal codons cannot be stop-codons
- And a lot more ...

```

Length of genome1: 1852441 (1852441)
Length of genome2: 2211485 (2211485)
Length of genome3: 2499279 (2499279)
Length of genome4: 1796846 (1796846)
Length of genome5: 2685015 (2685015)
Length of genome6: 2127839 (2127839)
Length of genome7: 2742531 (2742531)
Length of genome8: 2046115 (2046115)
Length of genome9: 2388435 (2388435)
Length of genome10: 1570485 (1570485)
Length of genome11: 2096309 (2096309)

```

Start-codon in normal genes:

```

ATG [8423, 'NCCC']
ATC [3, 'NCCC']
ATA [1, 'RCCC']
GTG [713, 'NCCC']
ATT [3, 'NCCC']
CTG [2, 'NCCC']
GTT [1, 'NCCC']
CTC [1, 'NCCC']
TTA [1, 'NCCC']
TTG [1020, 'NCCC']

```

Stop-codon in normal genes:

```

TAG [1949, 'CCCN']
TGA [1531, 'CCCN']
TAA [6686, 'CCCN']

```

Reversed stop-codon in reversed genes:

```

TTA (reverse-complement: TAA) [6596, 'NRRR']
CTA (reverse-complement: TAG) [2014, 'NRRR']
TCA (reverse-complement: TGA) [1148, 'NRRR']

```

Reversed start-codon in reversed genes:

```

TAT (reverse-complement: ATA) [2, 'RRRN']
ATG (reverse-complement: CAT) [1, 'RRRN']
GAT (reverse-complement: ATC) [1, 'RRRN']
CAT (reverse-complement: ATG) [8077, 'RRRN']
AAT (reverse-complement: ATT) [4, 'RRRN']
TAC (reverse-complement: GTA) [1, 'RRRN']
CAC (reverse-complement: GTG) [715, 'RRRN']
CAA (reverse-complement: TTG) [953, 'RRRN']
CAG (reverse-complement: CTG) [4, 'RRRN']

```

Summary

- We have considered the problem of selecting initial model parameters, and exemplified this problem by designing different HMMs for gene finding.
- Next time: See various extensions to the 'standard' HMM, which might be useful in some applications of HMMs, e.g. for gene finding.